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Contagion in a Core Periphery Financial Network with Heterogenous Banks

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Abstract

The goal of this paper is to analyze the propagation of idiosyncratic shocks through a financial network, identifying the relation between heterogeneity of institutions and the resilience of the system. I distinguish banks according to their size and degree of centrality in order to form a core-periphery network, similar to those empirically observed. Regarding the effects of unexpected shocks, I argue that connections work as a way of propagation of losses and prove the possibility of contagion in equilibrium. Unlike the intuitive perception, I point out that a gap between the size of central and peripheral agents is required for the first to achieve the expected systemic relevance. When it occurs, the presence of core-banks is crucial for easing the propagation of direct losses, as well as for protecting the system against peripheral shocks. I conclude by showing that there is a positive relation between the resilience of the core-periphery network and the degree of heterogeneity in the size of these agents.

Key Words: financial networks, degree of centrality, contagion, systemic risk.

JEL Codes: D85, G21, L14

[§] EPGE-FGV

1 - Introduction

The widespread financial system losses during the recent crisis have highlighted the need for a solid knowledge about risks of financial interconnections. The formation of a network, through interbank transactions, exposes financial institutions to one another even in the absence of directed link between them. Thus, idiosyncratic shocks are not restricted to directly affected banks or to their counterparts. In contrast, losses might spread throughout the financial network, so that individual shocks or in a small group of institutions potentially become a systemic event.

This paper analyzes how some features of financial networks affect the propagation of shocks through the system. More specifically, I develop a model that accounts for heterogeneity in size and centrality of banks and that relates the severity of contagion to these variables. The model is akin to Diamond and Dybvig (1983), in the sense of liquidity preferences shocks and provision of insurance by financial intermediaries. In line with Allen and Gale (2000), banks are divided in two regions, according to the received shock in the proportion of impatient depositors. While institutions of the same region suffer identical shocks, banks of different regions receive shocks that are negatively correlated. There is also an interbank market that enables the risk sharing between institutions and, as a consequence, the connection of the system.

The economy consists of ex ante homogenous regions that are formed by heterogenous banks. According to the centrality degree, institutions are classified as core or peripheral banks. While peripheral agents are only able to access the core bank of their region, the last might access all institutions, except the peripherals of the other region. This type of heterogeneity is suggested by empirical evidences in interbank markets and, for this reason, this pattern is adopted in the present work. For example, in Craig and Von Peter (2014), the authors argue that the German interbank market is centred in a small group of banks and that they are crucial to intermediate a large number of peripheral institutions. Evidences in this line are found in several countries[1].

Regarding interbank exposures, I assume that institutions hold the minimal deposits[2] needed to achieve the first-best[3], as in Allen and Gale (2000). To analyze financial fragility, the model is perturbed with the introduction of a state, in which an excess of aggregate demand for liquidity is verified.

Given the existence of instability, it is necessary to understand how direct contagion is related to centrality of the affected bank. The question is: are shocks in core banks more prone to infect direct counterparts? It would be intuitive that these institutions are more relevant to contagion, after all they have several interconnections. However, I prove exactly the opposite.

[1]For Germany, United Kingdom, Italy, Belgium and Netherlands, see Upper and Warms (2004), Wells (2004), Mistrulli (2011), Degryse and Nguyen (2007) and Van Lelyveld e In't Veld (2012), respectively. Evidences can also be found for Latin American countries. For Brazil and Colombia, see Tabak et al (2012) and Berndsen and Léon (2013).

[2]The concern is not to analyze the network formation, but the shock propagation and systemic risk in a exogenous network.

[3]I assume the absence of private markets, where consumers might trade in a unobservable manner. As initially pointed out by Jacklin (1987), the existence of private market might restrict the risk-sharing between agents and the achievement of first-best.

Diversification allows the losses to be better distributed across counterparts and, consequently, less harmful to each one. Thereby, considering homogenous banks in size, shocks in central agents are secondary in terms of direct contagion. In fact, it is interesting to show that a higher centrality is not sufficient to give them the intuitively expected systemic relevance. In the case of heterogeneity, the relation between centrality and the chance of direct contagion depends on the size of banks. When core banks are sufficiently large, they become the most relevant institutions for shocks' propagation.

The possibility of systemic contagion due to a shock in a core bank is also analyzed. I show that there is a threshold for the shock's size from which the domino effect is triggered and the entire network is affected. It would be interesting to verify if the same shock has such importance when it hits a peripheral agent. When core banks are sufficiently large, a peripheral shock is fairly restricted. The direct losses affect the central agent, but the spread is prevented. This result linked to several evidences of core banks' sizes[4] confirms the expected systemic importance of these institutions. It is worth noting that this result is produced by the interaction between their size and position in the network. While the first is responsible for increasing their ability to retain losses, the second consists in an obstacle to be overcome by the peripheral shock in order to hit the entire system. Thus, core banks tend to be the most systemically important banks in actual financial networks, for both the relative easy propagation of direct shocks and the protection of the system against peripheral ones.

This conclusion provides some implications for rescue policies. In highly heterogenous networks, rescue only makes sense[5] if the shock hits core banks directly. Therefore, it should be disregarded in case of peripheral shocks. Asymptotic properties are also analyzed and it can be shown that core-periphery networks never suffer contagion in this case.

It is worth noting that I consider shocks in the proportion of impatient depositors. Hence, the liquidity shortage depends on the affected bank's size. To check how my results are affected by this assumption, robustness tests are performed. When I consider shocks of fixed size, some thresholds are modified, however the main results remain unchanged, as shown in section 8.

This work is most related to Allen and Gale (2000). The liquidity shocks are assumed to be similar to those observed in the seminal paper, as well as the contagion mechanism, in which all depositors receive the same value per unit of deposit in case of default[6]. The main difference is the introduction of heterogeneity in financial institutions' centrality and size. As Freixas et al (2000), I assume the existence of banks that are more central in the network than others. In contrast, I take into account the presence of multiple cores in the network and the connection amongst them. In line with Iori et al (2006), I introduce heterogeneity in banks' size and allow for difference between banks' exposure, as well as in their liquidity buffers. My results differ from these papers, since centrality and size work together as natural barriers to contagion.

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[4]For example, in Craig and Von Peter (2014), the authors estimate the optimal core of German financial system and they argue that the institutions in this group are, in average, 51 times larger than banks in the periphery.

[5]I consider that the only objective of rescue policies is to avoid the contagion of the network.

[6]I do not consider lines and contingents payments (*sequential-service constraint*), as in Green and Lin (2003) and Peck and Shell (2003).

The paper is organized as follows. Section 2 presents the environment, while the optimal allocation is characterized in the following section. Contagion mechanism and consequences of heterogeneity are presented in section 4 and 5. Section 6 and 7 analyse contagion in equilibrium and the resilience of different networks, respectively. Robustness test and conclusion are presented in section 8 and 9.

2 - Environment

There are three dates: $t = 0, 1, 2$. The economy is composed by two ex ante identical regions, each one formed by $n+1$ banks. Bank i has a continuum of depositors (consumers) with measure d_i.

A consumer has endowment only at date 0 and it is equal to one unit of the single consumption good. I consider ex ante identical individuals who face the risk of being patient or impatient. An impatient consumer values consumption at date 1, while patient agents value consumption at date 2. As in Diamond and Dybvig (1983), individual's type is a private information, known in the second period. The probability of being impatient depends on the state of nature and the individual's region. More specifically, preferences are given by:

$$U(c_1, c_2) = \begin{cases} u(c_1), & \text{with probability } w(j, S); \\ u(c_2), & \text{with probability } 1 - w(j, S). \end{cases}$$

where c_t is the consumption at date $t = 1, 2$; $j \in \{A, B\}$ represents the individual's region and S, the state of nature. The utility function is assumed to be twice continuously differentiable, increasing, and strictly concave.

There are two equally likely states of nature, S_1 and S_2. The probability w might assume two values: w_L or w_H, where $0 < w_L < w_H < 1$. Table 1 shows how the probability of being an earlier consumer is related to individual's region and the state of nature.

State S\Region	A	B
S_1	w_H	w_L
S_2	w_L	w_H

Table 1 - Regional Liquidity Shocks

Turning to banks, I assume they do not have endowments and raise funds with consumers, through interbank deposits at $t = 0$. With resources in hand, banks might invest in two types of assets: liquid and illiquid. The last may be seen as a long-run asset that has a return $R > 1$ in the end of two periods. Although, it can be liquidated after one period, providing a return

$0 < r < 1$. On the other hand, the liquid asset takes only one period to achieve its maturity. Finally, I assume that only financial institutions have access to the long-run asset and, thereby, they have advantages over consumers in making investments.

Since depositors of different regions have negatively correlated liquidity shocks, there is always a region whose financial institutions have a liquidity demand excess and another with shortage. To eliminate this discrepancy, the interbank market is introduced. I assume, however, that banks cannot hold deposits in any financial institution. There are two groups of banks: peripheral and core-banks. More specifically, each region has one institution of the first type and n of the second. The difference between them is the access to the other institutions of the system. While a peripheral agent can only hold deposits in the core-bank of its region, the last institution might be counterpart of all peripheral banks of the same region and also the core-bank of the other region. Thus, core-banks possibly act as an intermediary in the interbank market.

3 - Optimal Risk Sharing and Interbank Deposits

In this section, I characterize the optimal allocation and the minimal interbank exposures which are capable for implementing the first-best. In this regard, I assume the existence of a central planner who makes the investment and consumption choices in order to maximize the unweighted sum of consumer's expected utility. Since individuals are ex ante identical, they are treated symmetrically. Thus, each impatient consumer receives c_1, while a patient individual receives c_2[7].

In the first period, the central planner chooses the amount allocated in the liquid and illiquid asset for each bank. For bank i, these variables are defined as y_i and x_i, respectively. Defining $\gamma \equiv \frac{w_H + w_L}{2}$, the central planner problem is[8]:

$$\max_{x_i, y_i} d_i[\gamma u(c_1) + (1-\gamma)u(c_2)]$$

$$s.a$$

$$x_i + y_i < d_i$$

$$d_i \gamma c_1 \leq y_i$$

$$d_i(1-\gamma)c_2 \leq Rx_i$$

[7]The optimal consumption allocation is independent of the state of nature, once there is no aggregate uncertainty.

[8]Clearly, the optimal way to provide consumption at date t is allocating the resources in the asset with maturity at the same date t. For this reason, I consider the feasibility constraints at date 1 and 2, as showed in the problem.

The restrictions above are the feasibility constraints at each date of the model. Solving the problem, the optimal allocation satisfies the first-order condition $u'(c_1) = Ru'(c_2)$. In turn, the optimal portfolio are given by:

$$y_i = d_i \gamma c_1 \quad e \quad x_i = \frac{d_i(1-\gamma)c_2}{R}$$

As proven in Allen and Gale (2000), the central planner can provide the first-best allocation, even if he cannot observe the consumer's type[9]. In order to do so, it is necessary to transfer resources between regions, since there is always one with a liquidity excess and another with shortage[10]. The first-best allocation might also be achieved in a decentralized market, through interbank deposits[11]. Note that interbank deposits work as insurance against liquidity shocks, thus allowing banks to attend their liquidity demand in every state. However, only core-banks have access to the neighboring region and, thus, such insurances might occur through them.

Considering c_1 and c_2 as the amount paid per unit of deposit at date 1 and 2 respectively, it is possible to show that there are several interbank deposits compatible with first-best allocation[12]. Since the contagion problem is intensified with larger cross holdings of deposits[13], I assume the minimal amount able to implement the first best-allocation. To formally characterize this structure, define B^j as the set of banks of region j, where $j \in \{A, B\}$ and B_0^j represents the core-bank of this region. Define z_{ik} as the total deposits held in bank k by bank i. Hence, taking $i \in B^j \setminus \{B_0^j\}$, the minimal deposits are given by:

$$z_{ik} = \begin{cases} (w_H - \gamma)d_i, & \text{se } k = B_0^j; \\ 0, & \text{c.c.} \end{cases} \tag{1}$$

On the other hand, core-banks must deposit the amount received from its peripheral counterparts and the value of its possible liquidity shortage in the other core-agent. Defining $-j$ as the neighbouring region of j, it follows that:

[9]The optimal allocation automatically satisfies the incentive constraints, since $u'(c_1) \geq u'(c_2)$ implies $c_1 \leq c_2$.

[10]To understand this point, define B^j as the set of banks of region $j \in \{A, B\}$. In the second period, when $S = S_1$ for example, region A has a liquidity demand excess of $(w_H - \gamma)c_1 \sum_{i \in B^A} d_i$, while region B has a supply excess of $(\gamma - w_L)c_1 \sum_{i \in B^B} d_i$. As the regions are identical regarding total wealth, the amount needed by one is exactly the surplus of the other and the central planner might achieve the first-best through transferences between regions.

[11]If interbank deposits are not allowed, the financial institutions are not able to provide the first-best allocation, since the feasibility constraint would not be satisfied in the second period for the banks of one region.

[12]The interbank deposits are compatible with first-best allocation when they pay c_1 and c_2 for impatient and patient consumers, respectively, and satisfy the constraints of central planner's problem for every bank and state of nature. Note that any structure of interbank deposits that fill the liquidity scarcity of the banks in need, without letting the others in lacking of resources, provide the first-best allocation.

[13]See Allen and Gale (2000).

$$z_{B_0^j k} = \begin{cases} 0, & \forall k \in B^{-j} \setminus \{B_0^{-j}\} \text{ ou } k = B_0^j; \\ (w_H - \gamma) \sum\limits_{i \in B^j} d_i, & \text{p/ } k = B_0^{-j}; \\ (w_H - \gamma) d_k, & \text{c.c.} \end{cases} \tag{2}$$

Assuming that core-banks choose to first withdraw from banks with liquidity excess when it is weakly optimal to do so, the optimal allocation is achieved through deposits defined in (1) and (2). Notice that the cross-holdings of deposits create a financial network, where banks are exposed to one another even if there is no direct link between them. The network created in this environment is called a core-periphery network and its representation follows in the figure below.

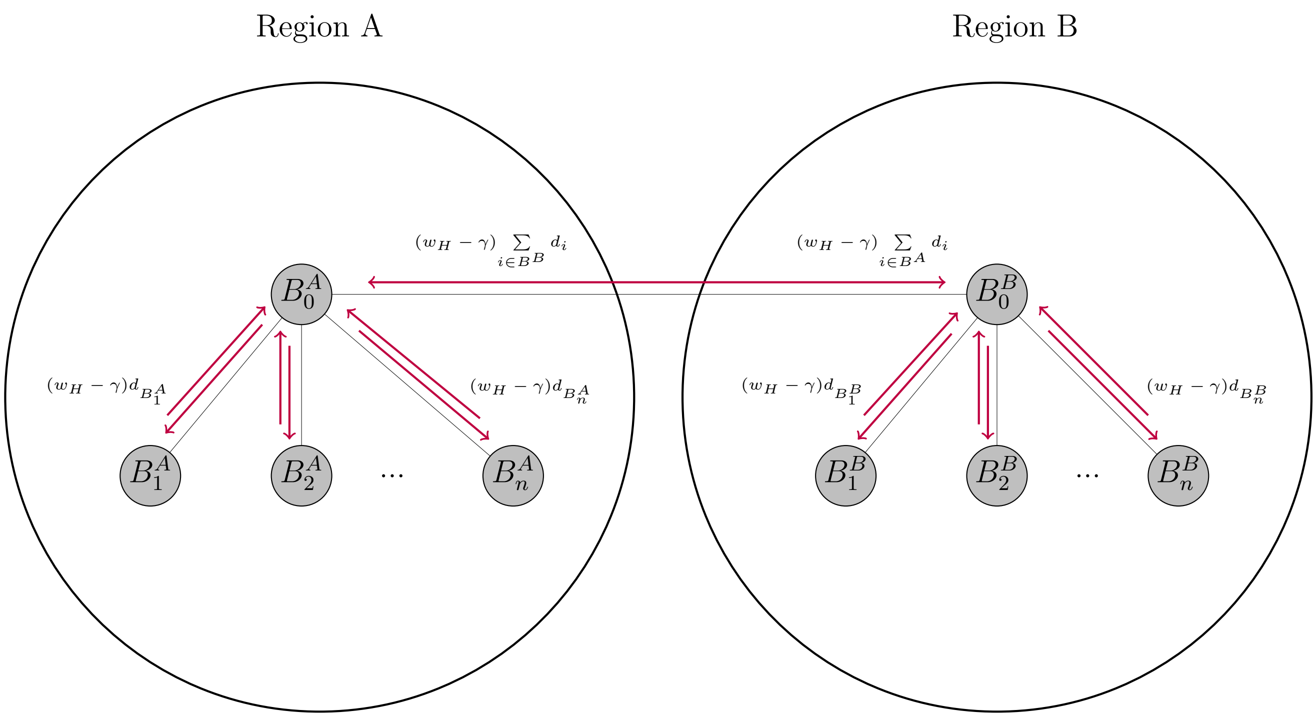

Figure 1: Core-Periphery Network - Minimal Exposures Compatible with Optimal Allocation.

4 - Fragility

Since the financial system is highly interconnected, its fragility is a major point of concern. In this environment, the network is stable[14] and interbank connections work as an efficient way to reallocate resources among regions. In contrast, if the aggregate demand for liquidity is greater than the supply, it is possible to verify financial instability and the links possibly work as a way of propagation of losses.

To study financial fragility, I perturb the model allowing the existence of a zero probability state, in which there is a liquidity demand excess. In this state, denominated S_3, every bank has a proportion γ of impatient depositors, except bank k. This institution suffers an additive shock of $\epsilon > 0$, i.e, $w_k = \gamma + \epsilon$. Once the probability assigned to that state in the first period is zero, the optimal portfolio and banks' cross holdings do not change when compared with the previous setting. In the next periods, however, the continuation equilibrium does not necessarily remain the same. When S_3 occurs, the optimal choices at date 1 and 2 are affected. In the rest of the paper, I analyse the continuation equilibrium when the state S_3 occurs, assuming the optimal allocation characterized in section III and the interbank deposits given by (1) and (2).

4.1 - Definition of Continuation Equilibrium

The continuation equilibrium is a subgame perfect Nash equilibrium, regarding dates 1 and 2, of banks' decisions about assets liquidation and withdrawals of interbank deposits; and consumers' decision about which period to withdraw their deposits; such that the contracts are enforced and consumers' utility maximized.

4.2 - Consumers

Consumers must decide whether to withdraw at date 1 or date 2. For impatient consumers, it is always optimum to withdraw at date 1, while patient individuals' choice depends on the amount received in each period. If the amount paid per unit of deposit at date 2 is lower than the value paid in the previous date, they prefer to withdraw early and invest these resources in the liquid asset. I assume that patient consumers always withdraw at date 2 if they consider that it is weakly preferable to do so. This assumption is adopted to avoid the existence of bank crisis, when banks are not actually insolvent.

4.3 - Banks: The Liquidation Pecking-Order and Bankruptcy

Banks must choose which assets liquidate for attending the liquidity demand of depositors at date 1. This decision basically involves the liquidation cost of each asset, that is, the cost

[14] In other words, there is no possibility of banks bankruptcy and contagion, consequently.

of obtaining consumption at date 1 in terms of consumption at date 2. Note that the three investment technologies have different liquidation costs and the relation between them defines the liquidation pecking-order.

Clearly, the cost of obtaining current consumption by liquidating the liquid asset is the lowest one[15]. Thus, banks first choose to liquidate this asset and, then, if it is necessary, they analyse the liquidation cost of the others. Considering usual utility functions[16] and assuming $-c_t u''(c_t)/u'(c_t) \geq 1$, the interbank deposit is the next asset to be liquidated[17]. Under these assumptions, the liquidation pecking-order is:

$$1 < \frac{c_2}{c_1} < \frac{R}{r}$$

In words, it means that banks optimally choose to first liquidate the liquid asset, then the interbank deposits and, in the last case, the illiquid asset. If financial institutions can attend the liquidity demand of depositors without withdrawing all their assets, they only liquidate the remaining amount at date 2. Otherwise, they go bankrupt and all depositors are treated equally.

4.4 - Contagion Mechanism

When an institution receives a shock, it possibly goes bankrupt and the losses may spread through the network. Since there is no region with liquidity excess in the state S_3, each bank totally uses their resources in the liquid asset in order to attend its depositors at date 1. However, these resources are not enough to attend the affected bank's shortage and this agent needs to withdraw at least some of its interbank deposits. The interaction between banks' optimal choices results in a process of mutual withdrawals, leading to a complete liquidation of interbank deposits in the system. At the end of this process, the liquidity shortage of bank k is not satisfied and its long-run asset must be liquidated.

Nonetheless, there is a maximum amount of consumption that can be obtained by liquidating the long-run asset without causing a bank-run. Banks must pay at least c_1 to patient consumers at date 2, otherwise they withdraw early and the institution suffers a run. Thus, the liquidity buffer of bank k is:

$$b_k(w) = r\left[x_k - \frac{d_k(1-w)c_1}{R}\right]$$

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[15] The liquidation cost of the short-run asset is one and it is lower than the others, since I am working with the optimal contracts given by first-order condition and the hypothesis of utility function's concavity.

[16] For example, the logarithmic function or the constant relative risk aversion utility function $\left(u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}\right)$

[17] For low levels of relative risk aversion, banks prefer to liquidate the illiquid asset before the interbank deposits.

Since its excess of liquidity demand is $\epsilon d_k c_1$, the contracted amount per unit of deposit is paid at date 1 if and only if $\epsilon d_k c_1 \leq b_k(\gamma + \epsilon)$. In this case, only late consumers are affected, receiving less than c_2 at date 2. Nevertheless, the amount paid is greater than c_1 and, therefore, there is no bank-run. On the other hand, if:

$$\epsilon d_k c_1 > b_k(\gamma + \epsilon), \tag{3}$$

patient individuals anticipate their withdrawals and the institution has to liquidate all its assets at date 1. Therefore, the payment per unit of deposit held in bank k is such that equates the value of its assets and liabilities.

$$q_k = \frac{y_k + r x_k + \sum_{i \in B^j \cup B^{-j}} z_{ki} q_i}{d_k + \sum\limits_{i \in B^j \cup B^{-j}} z_{ik}} \tag{4}$$

As $q_k \leq c_1$, the shock must result in loss for the direct counterparts of bank k. Thus, its counterparts might be affected even if they were not directly hit by the shock. Defining LGD_{kl} as the loss-given-default induced by bank k to bank l, the last one goes bankrupt by contagion if the loss absorbed is greater than its liquidity buffer. That is,

$$LGD_{kl} = z_{lk}(c_1 - q_k) > b_l(\gamma) \tag{5}$$

Finally, contagion is not restricted to banks directly exposed to the affected bank. Shocks might spread and affect institutions that are far from bank k in the network. However, that only happens if there is a path between them[18]. When a path between bank k and the other banks exists, the entire system might collapse as a result of an idiosyncratic shock.

5 - Heterogeneity and Contagion

In this section, I am willing to answer the following question: How do banks' characteristics affect the chance of contagion? Since financial institutions might differ according to their size (measure of depositors) and their position in the network, each feature is separately analysed.

[18] A link between banks i and j is represented by λ_{ij} and means that $i \in C_j$. A path between k e j is a sequence of links $\lambda_{12}, \lambda_{23}, ..., \lambda_{(M-1)M}$ such that $m \in C_{m+1}$ $\forall m \in \{1, ..., M-1\}$, with $1 = k$ and $M = j$.

Focusing on size, it is possible to note that both banks' exposure and liquidity buffer depend on it. In the case of a peripheral bank, these variables are linear functions of bank's size. Hence, if bank $k \in B^j \setminus \{B_0^j\}$ fails by contagion, a peripheral institution i, with $d_i \neq d_k$, defaults as well[19]. In contrast, it is not valid for core-banks. Once S_3 is a zero-probability state, its liquidity buffer depends only on its size, while its exposure is directly related to the total regions' size. Then, a higher measure of depositors implies in a higher resilience in that case.

Lemma 1. The resilience of peripheral banks does not depend on their size, while core-banks are more resilient when they get bigger.

Another point of interest is the relation between bank's size and its resilience to an exogenous shock. Since I am considering additive shocks in the proportion of impatient consumers, the total liquidity necessity is proportional to the affected bank's size, as well as its liquidity buffer. Thus, banks' size does not work as a protection against exogenous shocks. In other words, if $\epsilon > \underline{\epsilon}$ is able to cause a bank-run against an specific institution, then any bank goes bankrupt when directly hit by it. The threshold definition follows (6), as derived in the appendix. The assumption of additive shocks implies that the excess of liquidity demand varies according to the affected bank's size. This assumption is used throughout the paper, however I show in section 8 that the main results are maintained when shocks of fixed sizes are considered.

$$\underline{\epsilon} \equiv \frac{r}{R-r}(1-\gamma)\left[\frac{c_2}{c_1} - 1\right] \tag{6}$$

Financial institutions are also heterogeneous in their position in the network. According to this feature, there are two types of agents: core-banks and peripheral ones. As suggested by the nomenclature, financial institutions might have distinct levels of centrality. Analysing the degree centrality, defined as the ratio of the number of counterparts to the total possible, it is clear that core-banks are more central than peripheral institutions. The difference between banks' centrality is even higher when the measure of *Betweenness Centrality* is analysed. I.e, when it is considered how well situated an institution is in terms of its presence on the shortest paths between banks[20]. Thereby, core-banks are more central due both to its higher number of counterparts and its intermediation role.

Note that each feature mentioned above has its own effects on easing/blocking contagion. To see the degree centrality effect, suppose that bank k is hit by a shock $\epsilon > \underline{\epsilon}$. Clearly, it needs to liquidate an excessive amount of long-run assets in order to honor its agreements at date 1. Suppose further that k is peripheral, i.e, $k \in B^j \setminus \{B_0^j\}$ where $j \in \{A, B\}$. Using (5), the interbank deposits (1) and (2), the optimal contracts and assuming that only bank k goes bankrupt, the loss-given-default absorbed by the core-bank is:

[19] Clearly, institution i goes bankrupt if its counterpart defaults initially.

[20] Note that the network shown in figure 1 has only one shortest path between banks of different regions and core-banks are always present on it. In contrast, peripheral banks never lie on the shortest path.

$$LGD_{kB_0^j} = \frac{z_{B_0^j k}(1-\gamma)c_1}{1+w_H-\gamma}\left[1-\frac{c_2}{c_1}\frac{r}{R}\right]$$

On the other hand, if bank k is a central agent, the loss-given-default absorbed by a peripheral bank assumes the following expression.

$$LGD_{B_0^j i} = \frac{z_{iB_0^j}(1-\gamma)c_1 d_{B_0^j}}{d_{B_0^j} + (w_H-\gamma)\sum\limits_{i\in B^{-j}\cup B^j\setminus\{B_0^j\}} d_i}\left[1-\frac{c_2}{c_1}\frac{r}{R}\right]$$

Lemma 2. The loss-given-default induced by a peripheral bank to a core-bank is higher than the loss-given-default induced in the opposite case.

I prove that a peripheral bank induces higher losses to its creditor than a core-bank to its peripheral counterparts when they are directly hit by the shock $\epsilon > \underline{\epsilon}$. The proof follows in the appendix and does not depend on institutions' size[21]. The lemma is essentially driven by the higher number of core-bank's counterparts. Since peripheral banks have only one creditor, the losses are totally absorbed by it. In contrast, if a central agent goes bankrupt, its losses might be shared with several counterparts and, thus, are less harmful to each one.

In turn, the *betweenness centrality* effect goes in the opposite way in terms of easing/blocking contagion. Notice that the institution less exposed to a core-bank is counterpart of its counterpart. Therefore, the loss induced by the liquidation of a central agent directly hits $n+1$ banks and possibly the other n institutions through its counterparts. However, when a shock hits a peripheral agent, a larger path must be taken to affect the entire system and core-banks might block the spread in certain cases. Thereby, even though the loss-given-default induced by a core-bank is lower, it hits other banks more easily and, if sufficiently large, might affect the entire network.

Then, the two intrinsic features of centrality act in different directions on the chance of contagion occurrence. It is worth noting that the net effect of these forces does not always have the same sign. In other words, it is not necessarily true that a shock is more harmful when it hits a core-bank. There are circunstances in which its importance is even higher in the peripheral case. In order to understand this point, remember that, when a shock $\epsilon > \underline{\epsilon}$ affects a core-bank, the contagion of its counterparts occurs if (5) is respected. Note that both sides of the inequality depend on the return of long-run asset when prematurely liquidated (r). More specifically, the

[21]Note that both the liquidation value and the liability (at date 1) of a peripheral bank are proportional to its size. Thus, the amount paid by this agent per unit of deposit does not depend on its measure of depositors. Furthermore, the interbank exposure of a core-bank to its peripheral counterpart is proportional to the size of the later one, instead of its own size. Then, the ratio of banks' exposure does not depend on their size as well.

chance of contagion is negatively related to this parameter[22]. It would be interesting, then, to define the highest value of r that would be able to cause the failure of a peripheral bank when the shock hits a central agent. The definition of the threshold is given by:

$$r^N = \frac{R(w_H-\gamma)}{\left(\frac{c_2}{c_1}-1\right)\left(1+w_H-\gamma+(w_H-\gamma)\sum_{i\in B^j\cup B^{-j}\setminus\{B_0^j\cup B_0^{-j}\}}\frac{d_i}{d_{B_0^j}}\right)+(w_H-\gamma)\frac{c_2}{c_1}} \tag{7}$$

When the affected bank is peripheral, the threshold assumes the following expression:

$$r^P = \frac{R(w_H-\gamma)}{\frac{d_{B_0^j}}{d_k}\left(\frac{c_2}{c_1}-1\right)(1+w_H-\gamma)+(w_H-\gamma)\frac{c_2}{c_1}} \tag{8}$$

It should be noted that both thresholds are functions of the ratio of bank's size $(d_k/d_{B_0^j})$. The first is a decreasing and convex function, while the second one is increasing and concave. The functions follow in figure 3.

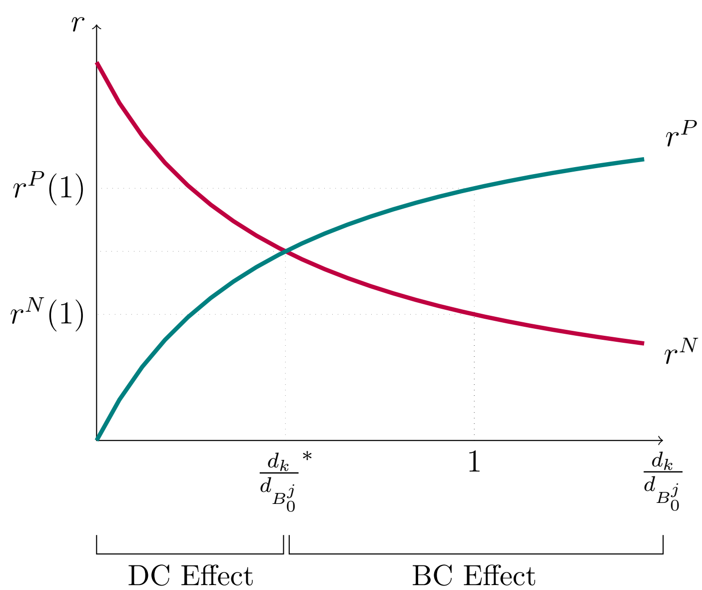

Figura 2: The threshold of r as a function of banks' size.

As it can be seen, there is a point $d_k/d_{B_0^j}{}^*$ after which a threshold is overcome by the other. Note that after this point the peripheral banks' threshold is larger than the one verified for core-banks. Surprisingly, in that case, a shock in a peripheral agent affects more easily the core-bank

[22]The return r affects positively the loss-given-default. Furthermore, it has a effect of lowering banks' liquidity buffer.

than the other way around. More specifically, if $d_k/d_{B_0^j} \in (d_k/d_{B_0^j}{}^*, \infty)$, the degree centrality effect prevails. In turn, the net effect is the opposite in the complementary interval. In addition, I prove that the ratio which equals the thresholds is lower than one. Since empirical evidences suggest that central agents are considerably bigger than the peripheral ones[23], it is reasonable to believe that the betweenness centrality effect prevails in the networks similar to those observed in the data. In a different manner, shocks in core-banks tend to be more relevant in terms of direct contagion in that case.

6 - Contagion in Equilibrium

Since shocks might spread throughout the network, another point of concern is the possibility of a systemic failure in equilibrium. More precisely, it should be analysed under which conditions the fixed point of the contagion problem is the set composed of all banks of the system[24]. I also analyse if such conditions are more restrictive according to the characteristics of the affected bank.

Proposition 1. Consider the market structure described in figure 1 and perturb the model by the addition of a zero-probability state S_3. Assume that bank $i \in N_j$ chooses an investment portfolio (x_i, y_i, z_i), where (x_i, y_i) is the first-best portfolio and $z_i = (z_{iI^j}, z_{iI^{-j}}, z_{i1}, z_{i2}, ..., z_{i2n})$ is formed by (1) and (2). Suppose that a core-bank k is hit by a shock $\epsilon > \underline{\epsilon}$. Then, if at least one institution of its region goes bankrupt by contagion, the entire system collapses.

Proof: Step 1: I initially argue that the affected bank induces non-zero losses to its creditors. Once S_3 has taken place, the interbank deposits are not enough to attend the core-bank's lack of liquidity. Therefore, the liquidation of its long-run asset is necessary. However, $\epsilon > \underline{\epsilon}$ and k suffers a bank-run. As a consequence, its assets must be completely liquidated at date 1. Since the feasibility constraint is not respected when $q_k = c_1$, the core-bank pays $q_k < c_1$ per unit of deposit.

$$(\gamma + \epsilon)d_k c_1 + (1 - \gamma - \epsilon)d_k c_1 > \gamma d_k c_1 + b_k(\gamma + \epsilon) + r\frac{(1 - \gamma - \epsilon)d_k c_1}{R} = y_k + r x_k$$

Step 2: Having established that the affected bank induces non-zero losses to its creditors, I next show that its direct counterparts fail by contagion. First, suppose that at least one institution of its region goes bankrupt. By lemma 1, it is clear that all the other peripheral agents fail as well, no matter their size.

[23]For example, Craig and Von Peter(2014) argues that in the German interbank market the average of core-banks' total assets is 51 times higher than the verified for peripheral banks.

[24]The algorithm used and the unicity/convergence of the solution follows in the appendix.

Furthermore, I show in the appendix that there is a threshold of payment (q_k) after which the counterparts do not suffer contagion. However, the thresholds assume different expressions according to the centrality of the creditor. Analysing them, it is possible to show that the peripheral counterparts' threshold is lower than the one verified in the case of a central creditor. It means that the central counterpart of a core-bank suffers contagion more easily than the others. Intuitively, this results is driven by the higher exposure of core-banks to each other, which is inherent to their intermediation role. Since I show that all peripheral counterparts go bankrupt, then the core-bank of the other region also fails.

Step 3: Finally, it must be proved that the other peripheral banks suffer contagion as well. Note that banks go bankrupt if and only if they cannot honor their agreements when the loss-given-default, induced by their counterpart, is compatible with the amount c_1 hypothetically paid by them. Once the direct peripheral counterparts initially fail, the result follows. ■

Proposition 1 states that under certain conditions a shock in a core-bank might lead to a systemic contagion in equilibrium. Financial contagion as an equilibrium phenomenon was first modeled in Allen and Gale (2000), when they considered a circular network structure composed of homogenous banks[25]. Note, however, that I prove a more general result, since the existence of heterogeneity in banks' size and centrality is allowed in this paper. Even more important, though, I show the possibility of a systemic contagion in a network similar to those observed in the data.

Moreover, the proposition highlights the systemic relevance of a shock $\epsilon > \underline{\epsilon}$ when it affects a core-bank. In addition, it would be interesting to verify if the same holds when the shock hits an agent with different centrality in the network. Proposition 2 affirms that, when the same shock affects a peripheral institution and the core-bank is sufficiently large, its systemic effects are substantially restricted. In contrast, if the core-bank is relatively small, the entire system fails by contagion.

Proposition 2. Consider the market structure described in figure 1 and perturb the model by the addition of a zero-probability state S_3. Assume that bank $i \in N_j$ chooses an investment portfolio (x_i, y_i, z_i), where (x_i, y_i) is the first-best portfolio and $z_i = (z_{iI^j}, z_{iI^{-j}}, z_{i1}, z_{i2}, ..., z_{i2n})$ is formed by (1) and (2). Assume the same parameters used in proposition 1 and suppose that a peripheral bank $k \in B^j \setminus \{B_0^j\}$ is hit by a shock $\epsilon > \underline{\epsilon}$. Then, there is a threshold $d^M_{B_0^j}$ such that if $d_{B_0^j} \geq d^M_{B_0^j}$, the core-bank totally absorbs the losses induced by k and protect the system against contagion.

$$d^M_{B_0^j} \equiv d_k \frac{(w_H - \gamma)c_1}{(1 + w_H - \gamma)(c_2 - c_1)} \left[\frac{R}{r} - \frac{c_2}{c_1}\right] \equiv d_k A \tag{9}$$

[25]In the incomplete network structure studied by Allen and Gale (2000), bank i is only exposed to bank $i+1$, whose liquidity shocks are negatively correlated to its shocks.

Proof: Step 1: Since $\epsilon > \underline{\epsilon}$, bank k goes bankrupt. Initially, it should be shown that $d^M_{B^j_0}$ is in fact a threshold. Note that the central agent fails by contagion if and only if it is not able to honor its agreements when only bank k do not pay c_1 per unit of deposit. Thus, using (5), the threshold is determined. Since only the right hand side of (5) depends on core-bank's size and this relation is strictly increasing, (9) defines in fact a threshold[26].

Step 2: In this step, I show that the entire system collapses when $d_{B^j_0} < d^M_{B^j_0}$. First, the contagion of core-banks' counterparts is analysed. Once k goes bankrupt, it does not pay c_1 per unit of deposit. Consequently, the loss-given-default induced by the central agent to its counterparts is greater than in the case when it is directly hit by the shock, as in proposition 1. Then, the contagion of core-bank's creditors follows. Finally, I argue that all peripheral banks of the other region default as well. Since the regions have the same size, this point is clearly true, whereas the peripheral banks of the affected region become insolvent when only bank k and the respective core-bank do not honor its agreements. ■

Thereby, the shock might be systemically more relevant according to the affected bank's centrality. It is worth noting that core-banks are crucial both for spreading losses in case of direct default and for protecting the system against peripheral shocks. In proposition 2, when a peripheral bank is hit, these agents may act as a barrier, preventing the propagation of losses throughout the system. The protective role of core-banks is due to both their position in the network and their size. The former is significant, since peripheral shocks might cause the failure of a core-bank to hit the rest of the system. In turn, the role of size is related to core-banks' capacity to absorb losses. Intuitively, their position might be understood as a natural barrier to be overcome, while their size should be seen as the strength of this barrier. Thus, when sufficiently large, core-banks might be crucial to avoid the spread of peripheral shocks.

Another key point for contagion occurrence is the size of the affected bank. Turning back to (9), it can be seen that A is greater than zero and, therefore, the threshold is directly related to the affected bank's size. Thus, if a core-bank is able to avoid the propagation of losses caused by the failure of the largest peripheral institution, the system never collapses when a shock hits the periphery. In addition, I show that A is greater than one. Consequently, core-banks should be the largest institutions in the network for protecting the system against any peripheral shock. Combining proposition 1 and 2, it can be concluded that, when core banks are sufficiently large, the same shock is able to cause a systemic collapse, as well as only the bankruptcy of the affected bank. In addition to highlighting the systemic relevance of core-banks, this conclusion has implications for rescue policies. Note that, in this case, rescue policies only make sense if the shock hits a core-bank[27].

Corollary 1. Assume that $d_{B^j_0} \geq (\max_{i \in B^j \setminus \{B^j_0\}} d_i)A$. Then, the system only collapses when a shock hits a central agent in the network.

[26] One may argue that the absorbed loss implicit in (9) is not the actual one when the core-bank's size is lower than $d^M_{B^j_0}$. In fact, it is the lower-bound on the actual loss. Then, the central agent must suffer contagion in that case.

[27] I am considering that the only goal of rescue policies is to avoid contagion.

7 - Resilience

As shown before, a financial system might suffer contagion. Intuitively, the resilience of a financial network should be related to its characteristics, for example: its structure, number of banks and the heterogeneity between banks' size. Aiming to compare the resilience of some networks, I analyse in this section how networks' features affect contagion occurrence.

Throughout this paper, it can be noted that banks' size heterogeneity has non-trivial role for blocking contagion. According to empirical evidences, for example in Craig and Von Peter (2014)[28] and Fricke and Lux (2012)[29], the largest banks of the network belong in general to the system's core. Then, considering that core-banks are the largest agents in the network, I analyse how differences in institutions' size affect the possibility of contagion. In case of a peripheral shock, contagion is more easily avoided when the size differential increases. However, if the affected institution is a core-bank, the failure of peripheral agents takes place more frequently and the opposite is verified for the central counterpart. To understand this point, note that the payment capacity of a core-bank is an increasing function of banks' size differential. The value of its assets decreases proportionally more than its liabilities, due to the lower relative contribution of the payments done by its counterparts[30]. The negative effect on its assets' value is harmful to all its counterparts. However, in the case of a central counterpart, there is also an opposite force: the increase in the ratio of liquidity buffer to bank exposure. Considering the different effects in this case, I show that the positive one prevails[31]. Therefore, according to the centrality of the affected bank, an higher heterogeneity might ease or block the direct contagion. Actually, it blocks contagion in case of a shock in a peripheral institution, while the net effect depends on the counterpart's position when a shock hits a core-bank. Assuming, however, that banks are equally susceptible to exogenous shocks, lemma 3 follows below.

Lema 3. Core-periphery financial networks are more resilient, when the size heterogeneity between core-banks and peripheral banks increases.

Intuitively, one should also expect the existence of a relation between the total number of peripheral institutions and the resilience of a core-periphery network. Clearly, when a shock hits a peripheral agent, the loss-given-default is the same regardless the number of peripheral institutions in the network. For a core-bank, although, the payment capacity per unit of deposit in case of default is an increasing function of the number of its counterparts. Therefore, there is a reduction of the loss induced by core-banks when n increases. In order to evaluate the

[28]The paper develops a theoretical structure for a core-periphery network and estimates the optimal core, so that the distance between this structure and the German network is minimized. Having obtained the optimal core, the authors create a binary variable, for each bank, which assumes the unit value if an institution belongs to the optimal core and zero, otherwise. Using Maximum likelihood methods, the binary variable is regressed against total assets, for example. The estimated coefficients are highly significants.

[29]Following the same line used by Craig and Von Peter (2014), the work studies the Italian interbank system.

[30]I consider the case in which the counterparts pay c_1 per unit of deposit withdrawn at date 1. To better understand, see the algorithm used in equilibrium determination, presented in the appendix

[31]I am studying the direct effects of size heterogeneity, instead of accounting the indirect consequences.

total loss, however, it is necessary to study the exposure of each counterpart. As it can be seen in (1) and (2), only core-banks' deposits held in one another depend on n. Hence, when the periphery gets bigger, there are two effects on direct contagion of the central counterpart. While the greater payment capacity works as an obstacle, the higher bank exposure works in the other way. However, it can be shown that the first effect prevails and, consequently, there is a positive relation between the resilience of a core-periphery network and the number of peripheral banks in each region.

Lemma 4. Consider identical core-periphery networks, except by the number of peripheral banks. The more resilient system is the one with higher peripheries, regarding the number of banks.

Having established the lemma above, a natural question arises. Is contagion a possible phenomenon regardless the size of n? Determining exactly the threshold would be complicated due to the several interactions of payments in equilibrium. However, asymptotic proprieties may be analysed. Initially, it is necessary to understand how a shock $\epsilon > \underline{\epsilon}$ in a core-bank affects the system. When n tends to infinity, the losses induced by the core-bank are well distributed and the payment per unit of deposit tends to the one which was initially promised. Thus, peripheral banks do not suffer contagion. The central counterpart in turn might fail as a result of a exposure which tends to explode. Actually, its contagion depends on the parameters. Studying the worst case, i.e, when both core-banks go bankrupt, the periphery does not fail by contagion. To conclude the analysis, it is needed to observe the propagation of losses when the shock hits a peripheral agent. Assuming that core-bank's size is lower than the threshold presented in proposition 2, the loss induced by the central agent tends to zero and its peripheral counterparts do not default on their agreements. Although, I show that if a core-bank suffers direct contagion, the other central agent fails as well. Nevertheless, its payments per unit of deposit still tends to the contracted value and peripheral banks do not go bankrupt. The formal demonstration might be seen in the appendix.

Lema 5. The core-periphery network is not asymptotically susceptible to contagion.

To conclude the analysis of relative resilience, it is worth comparing the stability of a core-periphery network to the one recurrently used in the literature, known as circular or incomplete network. So, assume homogeneity of bank's size and consider that the only difference between the two systems is their structure. Suppose now that $\epsilon > \underline{\epsilon}$ hits an institution in the circular network and its counterpart goes bankrupt. Allen and Gale (2000) shows that the entire system collapses in that case. Assuming the same parameters, this result is not necessarily found in a core-periphery network. Note that, in the circular network, all banks have only one creditor. Therefore, the losses induced by them is similar to the ones caused by the failure of a peripheral institution in a core-periphery network. The similarity is also verified for the threshold of r, before which there is direct contagion of the central counterpart. Turning to figure 2, it is possible to see that, when banks are homogenous in their size, the threshold for a core-bank is lower than the one for a peripheral bank. Thus, the existence of a direct contagion in the circular system does not imply the same result in a core-periphery network. Even if the shock hits a peripheral agent, contagion might not occur. Considering n sufficiently large, a core-periphery structure does not suffer contagion.

Reversing the exercise, assume that the core-periphery system collapses as a result of a shock $\epsilon > \underline{\epsilon}$. Then, the contagion of the circular network necessarily occurs when the same shock hits this system. Since the number of banks' creditors in the last network is less or equal to the one observed in the first structure, the induced loss is greater or equal in that case. Finally, note that the number of banks is unable to avoid the systemic contagion in a circular network, once the number of creditors is the same regardless the total amount of banks in the network.

Lema 6. Considering a sufficiently large number of banks, the circular network is less resilient than the core-periphery network.

8 - Robustness

As highlighted in section 5, the existence of banks' size heterogeneity implies that the aggregate excess of liquidity varies according to the affected bank's size. In this section, I show that my results rely weakly on this assumption.

With this purpose in mind, assume that a shock of fixed size hits a financial institution in the state S_3. Three shocks of this type might be considered: loss of short-run assets, destruction of long-run assets or additional costs. The first one was excluded, since the largest possible loss would not be able to break a bank. The second in turn is not very useful, once the shock would possibly be limited to a negligible size. Clearly, the maximum destruction of the long-run asset that a bank may face is the total value held on it. Then, in order to have homogenous shocks, they would be limited to the biggest possible loss that the smallest bank in the network can absorb[32]. Depending on the variance of banks' size, the non-trivial interval to contagion analysis might be not reached. Since shocks of additional costs might be used and their size is not restricted, I choose to use this class of shocks for testing my results.

Suppose, then, a financial institution, k, has to pay a cost of ϵ^F in the state S_3. Consider that this cost is a senior debt paid to an external agent. This shock causes a bank failure if and only if it is bigger than bank k's liquidity buffer. Since this variable depends on bank's size, the threshold also depends. Thus, in contrast to what happens in the case of heterogenous shocks, the same additional cost might lead some institution to the failure and not necessarily the others.

It is crucial to note that, once bank k goes bankrupt, systemic contagion does not depend on the type of the shock[33]. Hence, if a proportional shock may cause a systemic collapse, an

[32]More specifically, the shocks would be limited to be less than $\min_{i \in N} d_i \frac{(1-\gamma)c_2}{R}$

[33]To see this point, turn to (5) and note that q_K does not depend on the shock, as well as the loss-given-default induced by k

additive shock do the same when it is able to cause the initial failure. Assume that a core-bank is hit by a shock ϵ^F, such that:

$$\epsilon^F > \underline{\epsilon}^F \equiv \max_{i \in N} \underline{\epsilon}_i{}^F = \max_{i \in N} d_i \frac{(1-\gamma) r c_1}{R} \left[\frac{c_2}{c_1} - 1 \right], \tag{10}$$

then, if at least one institution of its region fails by contagion, the system collapses as well. Therefore, the main point of proposition 1 remains unchanged, when considering shocks of fixed size. The same is verified for proposition 2 and corollary 1.

Regarding resilience results, some considerations should be done. Considering lemma 3, note that, in case of additive shocks, the way in which size heterogeneity increases is relevant for my results. When peripheral banks' size reduces, it is not necessarily true that there is a positive relation between size heterogeneity and network resilience[34]. In this case, lemma 3 is still valid if the reduction of peripheral banks' size is sufficiently large such that the threshold in (9) is reached. In addition, when considering homogenous shocks, lemma 4 does not necessarily hold. Actually, it should only account for contagion given a initial failure, instead of network resilience.

To conclude, it is necessary to highlight that lemma 5 and 6 remain unchanged. Thus, the main results of this paper does not rely on the assumption of proportional shocks. In other words, the key results are maintained when considering shocks of fixed size.

9 - Conclusion

As highlighted through the paper, a financial network is formed by interbank exposures. These connections act as a protection against expected shocks, while work as a way of propagation of losses in case of unexpected ones. Since there are empirical evidences that interbank markets resemble to a core-periphery network, I analyse the contagion issue considering this specific structure.

Allowing size and centrality heterogeneity, it is possible to see that the direct contagion is related to the characteristics of the institution initially bankrupt. I show that an higher centrality may not ease the spread of shocks. At the same time the failure of a core-bank might directly affect an higher number of counterparts, its capacity of loss mitigation makes this event less harmful to each counterpart. Additionally, it was shown that banks' size and centrality interact together determining the importance of a shock in each institution. I prove that, when the measure of depositors of a peripheral bank is greater or equal to the one verified in a core-bank, the direct contagion occurs more easily when a shock hits the less central institution. The opposite occurs when core-banks are sufficiently large. Since there are empirical evidences

[34]The reduction of peripheral banks' size eases the failure caused by an exogenous shocks, since it is reasonable that small shocks occurs more frequently.

that the central institutions are significatively bigger than the others, the centrality effect in the direction of blocking contagion seems to prevail in networks similar to the actual ones.

Extending the analysis, the contagion of indirect counterparts is also studied. I show that, under certain conditions, a shock in a core-bank might cause a systemic collapse. The contribution of this result is proving the possibility of contagion in equilibrium in a heterogenous network, regarding bank's size and centrality. More important, though, I prove the existence of systemic risk in a financial network similar to those observed in several countries.

In addition, the paper shows that the same shock does not necessarily have such dimension, when it affects a peripheral institution. If core-banks are sufficiently large, the effects of peripheral shocks are considerably restricted. Then, my results highlight the systemic importance of core-banks for both propagating shocks that directly affect them and protecting the system against peripheral failures. I also show that, when core-banks are sufficiently large, shocks are only able to cause a systemic contagion when they directly hit those central agents.

The relative resilience of some distinct networks is also analysed. Regarding this subject, the main result is achieved by comparing the core-periphery network to the circular one. It has been shown that the former is more resilient than the second one, implying that the systemic risk could be being overestimated in the literature.

Finally, the literature of financial networks is relatively recent and there are still several issues to be analysed. This paper studies contagion in a fixed network compatible with the first-best allocation and it is clear the necessity of a deeper understanding of network formation. Specially, explaining the forces behind the predominance of core-periphery networks in the interbank market of several countries and studying possible policies to undermine systemic risks in that case.

Appendix

1) Proof of Lemma 2

$$1 + (w_H - \gamma) < 1 + (w_H - \gamma)\frac{d_{B_0^{-j}}}{d_{B_0^j}} + (w_H - \gamma) \sum_{i \in B^j \cup B^{-j} \setminus \{B_0^j, B_0^{-j}\}} \frac{d_i}{d_{B_0^j}}$$

$$1 + (w_H - \gamma) < 1 + (w_H - \gamma) \sum_{i \in B^j \cup B^{-j} \setminus \{B_0^j\}} \frac{d_i}{d_{B_0^j}}$$

$$\frac{1}{1 + (w_H - \gamma)} > \frac{d_{B_0^j}}{d_{B_0^j} + (w_H - \gamma) \sum\limits_{i \in B^j \cup B^{-j} \setminus \{B_0^j\}} d_i}$$

Since $z_{B_0^j i} = z_{i B_0^j}$, then:

$$LGD_{iB_0^j} = \frac{z_{B_0^j i}}{1 + (w_H - \gamma)} > \frac{z_{iB_0^j} d_{B_0^j}}{d_{B_0^j} + (w_H - \gamma) \sum\limits_{i \in B^j \cup B^{-j} \setminus \{B_0^j\}} d_i} = LDG_{B_0^j i}$$

■

2) Threshold of the Shock

Suppose that ϵ is sufficiently large, such that bank k goes bankrupt. I.e,

$$\epsilon d_k c_1 > b_k(\gamma + \epsilon) = r\left[x_k - \frac{(1 - \gamma - \epsilon)d_k c_1}{R}\right] = \frac{r}{R}\left[(1 - \gamma)d_k c_2 - (1 - \gamma - \epsilon)d_k c_1\right]$$

$$\epsilon > \frac{r}{R - r}(1 - \gamma)\left[\frac{c_2}{c_1} - 1\right] \equiv \underline{\epsilon}$$

$$\epsilon d_i c_1 > \frac{r}{R - r}\left[(1 - \gamma)d_i c_2 - (1 - \gamma)d_i c_1\right] = b_i(\gamma + \epsilon)$$

Therefore, the shock $\epsilon > \underline{\epsilon}$ is able to cause the failure of any bank of the system.

$\blacksquare$

3) Algorithm - Equilibrium Determination

The algorithm used for determining the insolvent banks in equilibrium follows below.

i) Consider that only bank k goes bankrupt as a result of a exogenous shock and compute the liquidation value of its assets.

ii) Compute the LDG_{kj} $\forall j \in N$ and determine $D_k^1 = \{j \in N; LDG_{kj} > b_j(\gamma)\}$. If $D_k^1 = \varnothing$, only bank k does not honor its agreements and the other banks pay c_1 per unit of deposit withdrawn at date 1.

iii) If $D_k^1 \neq \varnothing$, then assume that only its elements and bank k go bankrupt. Compute the liquidation value of their assets.

iv) Determine $D_k^2 = \{j \in N; \sum\limits_{i \in D_k^1 \cup k} LDG_{ij} > b_j(\gamma)\}$. If $D_k^2 = \varnothing$, then D_k^1 is the set of insolvent banks in equilibrium.

v) If $D_k^2 \neq \varnothing$, then continue this process until the set of banks is exhausted or any other bank goes bankrupt.

4) Unicity of the Set of Insolvent Banks in Equilibrium

In the case of $\epsilon \leq \underline{\epsilon}$, the set of insolvent banks in equilibrium is empty and, trivially, unique. Then, assume that a shock $\epsilon > \underline{\epsilon}$ hits bank k. Suppose for contradiction the existence of multiple sets of insolvent banks in equilibrium, denominated $D_{k,1}^*, D_{k,2}^*, ..., D_{k,\theta}^*$. Note that k belongs to all these sets. However, since $D_{k,1}^* \neq D_{k,2}^* \neq ... \neq D_{k,\theta}^*$, for each possible pair of sets, there is an element that belongs to one and not to the other. Consider, for example: $D_{k,1}^*$ and $D_{k,2}^*$. Then, it is possible to say that there is an institution i_0 which belongs to one and not to the other. Without loss of generality, suppose that $i_0 \in D_{k,1}^*$ and $i_0 \notin D_{k,2}^*$. Since $i_0 \in D_{k,1}^*$, it is not able to pay c_1 per unit of deposit when the other banks of this set also go bankrupt. I.e:

$$\sum_{l \in D_{k,1}^* \backslash \{i_0\}} LGD_{l,i_0} > b_{i_0}(\gamma)$$

However, $i_0 \notin D^*_{k,2}$. It means that the loss suffered by it, due to the bankruptcy of all elements of $D^*_{k,2}$, is not large enough to cause its failure. Therefore, the inequality below is satisfied.

$$\sum_{l \in D^*_{k,2}} LGD_{l,i_0} < b_{i_0}(\gamma)$$

Consequently,

$$\sum_{l \in D^*_{k,1} \setminus \{i_0\}} LGD_{l,i_0} > \sum_{l \in D^*_{k,2}} LGD_{l,i_0}$$

Since the inequality above holds, $D^*_{k,1} \setminus \{i_0\} \neq D^*_{k,2}$. Actually, it is possible to say that $D^*_{k,1} \setminus \{i_0\} \not\subset D^*_{k,2}$, otherwise i_0 would also belong to $D^*_{k,2}$. This affirmative is true, once the total loss induced to i_0 increases or at least remains the same when an additional bank goes bankrupt. To understand this point, take $j \in D^*_{k,2} \setminus D^*_{k,1}$ and analyse the loss induced to i_0 when j is added to the set $D^*_{k,1} \setminus \{i_0\}$. Then,

$$\begin{aligned} \sum_{l \in (D^*_{k,1} \setminus i_0) \cup \{j\}} LGD_{l,i_0} = \sum_{l \in D^*_{k,1} \setminus \{i_0\}} z_{i_0 l} & \left[\frac{(1-\gamma) d_l c_1 \left(1 - \frac{r}{R}\frac{c_2}{c_1}\right) + \sum\limits_{i \in C_l} z_{li}(c_1 - q_i(q_{-i}))}{d_l + \sum\limits_{i; l \in C_l} z_{il}} \right] + \\ + z_{i_0 j} & \left[\frac{(1-\gamma) d_j c_1 \left(1 - \frac{r}{R}\frac{c_2}{c_1}\right) + \sum\limits_{i \in C_j} z_{ji}(c_1 - q_i(q_{-i}))}{d_j + \sum\limits_{i; j \in C_i} z_{ij}} \right] \end{aligned} \tag{11}$$

There are two possible cases: j is a counterpart of i_0 or not. When j is not a counterpart of i_0, the second term of the equation above is zero. Note that the first term never decreases when an additional bank goes bankrupt. It occurs because the payment done by insolvent institutions depends positively on the amount received by their deposits. Since I consider that bank j also defaults ($q_j < c_1$), then its counterparts are negatively affected. Perceive that they might not be counterparts [35]of i_0 and, if it is not the case, the last ones could be solvent. Then, the first term of the equation could remain the same or increase when an additional bank goes bankrupt. However, when j is counterpart of i_0, it is necessarily verified an increase of the loss absorbed by i_0, once the second term of the equation is positive.

[35]If j is not counterpart of a counterpart of i_0, it could be counterpart of a counterpart of a counterpart of i_0 and could affect it more indirectly. Note that the reasoning used for concluding the non-negative change of the first term might be extended and be used for distant paths between i_0 and j.

Therefore, $D^*_{k,1} \setminus \{i_0\} \not\subset D^*_{k,2}$, implying that $\exists\ i_1 \in D^*_{k,1} \setminus \{i_0\}$ such that $i_1 \notin D^*_{k,2}$. Thus,

$$\sum_{l \in D^*_{k,1} \setminus \{i_0, i_1\}} LGD_{l,i_1} > b_{i_1}(\gamma) \quad and \quad \sum_{l \in D^*_{k,2}} LGD_{l,i_1} < b_{i_1}(\gamma)$$

Once again, the inequalities above imply that $D^*_{k,1} \setminus \{i_0, i_1\} \not\subset D^*_{k,2}$. Note that the number of elements of $D_{k,1}$ is lower than the number of banks in the economy and, therefore, finite. Thus, repeating this process, it will be possible to find in some moment $\eta \in \mathbb{Z}$ such that $i_\eta \in D^*_{k,1} \setminus \{i_0, i_1, ..., i_{\eta-1}\} = \{k\}$ and $i_\eta \notin D^*_{k,2}$. Then, $k \notin D^*_{k,2}$.

■

5) Proof of Proposition 1

Step 1: In the body of the work.

Step 2: Suppose that at least institution $k \in B^j \setminus \{B^j_0\}$ goes bankrupt by contagion. Then, bank k fails when considering that only the core-bank does not honor its agreements. That is,

$$z_{kB^j_0}(c_1 - q_{B^j_0}) > b_k(\gamma) \tag{12}$$

where:

$$q_{B^j_0} = \frac{\gamma d_{B^j_0} c_1 + \frac{r}{R}(1-\gamma) d_{B^j_0} c_2 + (w_H - \gamma) c_1 (\sum_{i \in B^j \setminus \{B^j_0\}} d_i + \sum_{i \in B^j} d_i)}{d_{B^j_0} + (w_H - \gamma) \sum\limits_{i \in B^j \cup B^{-j} \setminus \{B^j_0\}} d_i}$$

By lemma 1, $\forall i \in B^j \setminus \{B^j_0\}$, the following inequality is valid: $LGD_{B^j_0 i} > b_i(\gamma)$. Thus,

$$q_{B^j_0} < q^{DP}_{B^j_0} \equiv c_1 \left[1 - \frac{r}{R} \frac{(1-\gamma)}{(w_H - \gamma)} \left(\frac{c_2}{c_1} - 1\right)\right]$$

It is needed to be shown that the same is verified in the case of a central counterpart. So, it is necessary that $LGD_{B^j_0 B^{-j}_0} > b_{B^{-j}_0}(\gamma)$. In other words, the inequality below must hold.

$$q_{B_0^j} < q_{B_0^j}^{DN} \equiv c_1 \left[1 - \left(\frac{d_{B_0^{-j}}}{\sum\limits_{i \in B^{-j}} d_i} \right) \frac{r}{R} \frac{(1-\gamma)}{(w_H - \gamma)} \left(\frac{c_2}{c_1} - 1 \right) \right]$$

Note that $q_{B_0^j}^{DP} < q_{B_0^j}^{DN}$. It means that if the payment done by the affected core-bank is sufficiently small to cause the failure a peripheral bank, then the central counterpart suffers contagion as well. Since $q_{I_j} < q_{B_0^j}^{DP}$, then the direct contagion of all counterparts occurs.

Step 3: I still need to prove that peripheral banks of the other region suffers indirect contagion. Thus, $\forall i \in B^{-j} \setminus \{B_0^{-j}\}$, it is necessary that :

$$z_{iB_0^{-j}}(c_1 - q_{B_0^{-j}}) < b_i(\gamma) \tag{13}$$

where:

$$q_{B_0^{-j}} = \frac{\gamma d_{B_0^{-j}} c_1 + \frac{r}{R}(1-\gamma) d_{B_0^{-j}} c_2 + (w_H - \gamma)(\sum_{i \in B^{-j} \setminus \{B_0^{-j}\}} d_i c_1 + \sum_{i \in B^{-j}} d_i q'_{B_0^j})}{d_{B_0^{-j}} + (w_H - \gamma) \sum\limits_{i \in B^{-j} \cup B_j \setminus \{B_0^{-j}\}} d_i}$$

Since the regions are identical and the core-bank of region j goes bankrupt, $q_{B_0^{-j}}$ in (13) is lower than $q_{B_0^j}$ in (12). In addition, $\forall i \in B^{-j}$, there is $l \in B^j$ such that $d_l = d_k$. Then:

$$z_{iB_0^{-j}}(c_1 - q_{B_0^{-j}}) > z_{lB_0^j}(c_1 - q_{B_0^j}) > b_l(\gamma) = b_i(\gamma),$$

Consequently, (13) is not satisfied and all peripheral banks of the neighbor region suffer contagion.

■

6) Proof of Proposition 2

Step 1: First, it is necessary to prove that there is no direct contagion when $d_{B_0^j} \geq d_{B_0^j}^M$. Consider initially that only bank k defaults. So, the payment per unit of deposit is given by:

$$\bar{q}_k = \frac{\gamma c_1 + \frac{r}{R}(1-\gamma)c_2 + (w_H - \gamma)c_1}{1 + (w_H - \gamma)}$$

Then, it is possible to write the loss-given-default of k as follows below. Since $d_{B_0^j} \geq d_{B_0^j}^M$, the core-bank does not fail and avoids the propagation of the shock.

$$LGD_{kB_0^j} = d_k \frac{(w_H - \gamma)(1-\gamma)c_1}{1 + w_H - \gamma}\left[1 - \frac{r}{R}\frac{c_2}{c_1}\right] = d_{B_0^j}^M (1-\gamma)c_1 \frac{r}{R}\left[\frac{c_2}{c_1} - 1\right] \leq b_{B_0^j}(\gamma)$$

Define now the lower bound on the loss-given-default of k as ($\underline{LGD}_{kB_0^j}$). Note that it occurs when only bank k goes bankrupt. Then, if $d_{B_0^j} < d_{B_0^j}^M$, the inequality below is satisfied and the core-bank suffers contagion.

$$b_{B_0^j}(\gamma) < d_{B_0^j}^M (1-\gamma)c_1 \frac{r}{R}\left[\frac{c_2}{c_1} - 1\right] = d_k \frac{(w_H - \gamma)(1-\gamma)c_1}{1 + w_H - \gamma}\left[1 - \frac{r}{R}\frac{c_2}{c_1}\right] = \underline{LGD}_{kB_0^j}$$

Step 2: Finally, I need to prove that, there is a systemic collapse if $d_{B_0^j} < d_{B_0^j}^M$. Suppose that only bank k and its direct counterpart do not pay c_1 per unit of deposit. Thus, the loss-given-default of the core-bank to an counterpart ι is given by $LGD_{B_0^j \iota} = z_{\iota B_0^j}(c_1 - q'_{B_0^j})$, where:

$$q'_{B_0^j} = \frac{\gamma d_{B_0^j} c_1 + \frac{r}{R}(1-\gamma) d_{B_0^j} c_2 + (w_H - \gamma)c_1 (\sum_{i \in B^j \setminus \{B_0^j, k\}} d_i + \sum_{i \in B^j} d_i) + (w_H - \gamma) d_k q_k}{d_{B_0^j} + (w_H - \gamma) \sum\limits_{i \in B^j \cup B^{-j} \setminus \{B_0^j\}} d_i}$$

Comparing the payment above to the one considered in (12) and noting that $q_k < c_1$, it is possible to conclude that $q_{B_0^j} > q'_{B_0^j}$. Since the parameters are the same of proposition 1, all peripheral banks of the same region suffer contagion.

Regarding the other region, the core-bank also goes bankrupt, since $q_{B_0^j}^{DP} < q_{B_0^j}^{DN}$ as shown in the proof of proposition 1. Then, supposing that only peripheral institution of region j and core-banks defaults, the payment per unit of deposit held in B_0^{-j} is given by:

$$q'_{B_0^{-j}} = \frac{\gamma d_{B_0^{-j}} c_1 + \frac{r}{R}(1-\gamma) d_{B_0^{-j}} c_2 + (w_H - \gamma)c_1 \sum_{i \in B^{-j} \setminus \{B_0^{-j}\}} d_i + (w_H - \gamma) q''_{B_0^j} \sum_{i \in B^{-j}} d_i}{d_{B_0^{-j}} + (w_H - \gamma) \sum\limits_{i \in B^{-j} \cup B_j \setminus \{B_0^{-j}\}} d_i}$$

Since (12) é satisfied and the regions are *ex-ante* identical, the entire network collapses.

■

7) The Threshold of Core-Bank's Size

Since the same parameters of proposition 1 are considered, it follows:

$$\frac{(w_H-\gamma)(1-\gamma)c_1 d_{B_0^j}}{d_{B_0^j}+(w_H-\gamma)\sum\limits_{i\in B^j\cup B^{-j}\setminus\{B_0^j\}} d_i}\left[1-\frac{c_2}{c_1}\frac{r}{R}\right]>(1-\gamma)c_1\frac{r}{R}\left[\frac{c_2}{c_1}-1\right]$$

Manipulating the inequality and dividing both side by $(1+w_H-\gamma)$, I show that A is bigger than one.

$$A\equiv\frac{(w_H-\gamma)c_1}{(1+w_H-\gamma)(c_2-c_1)}\left[\frac{R}{r}-\frac{c_2}{c_1}\right]>\left[1+\frac{(w_H-\gamma)}{(1+w_H-\gamma)}\sum_{i\in B^j\cup B^{-j}\setminus\{B_0^j,B_0^{-j}\}}\frac{d_i}{d_{B_0^j}}\right]>1$$

■

8) Proof Lemma 3

Suppose that bank k is hit by a shock $\epsilon>\underline{\epsilon}$.

Case 1: $k\in B^j\setminus\{B_0^j\}$

In order to have direct contagion, it is necessary that $LGD_{kB_0^j}>b_{B_0^j}(\gamma)$. I.e:

$$\frac{(w_H-\gamma)d_k\left[1-\frac{r}{R}\frac{c_2}{c_1}\right]}{1+w_H-\gamma}>d_{B_0^j}\frac{r}{R}\left[\frac{c_2}{c_1}-1\right]$$

An higher heterogeneity might occur as a result of an increase of core-bank's size, a decrease of peripheral bank's size or both. Then, the result clearly follows.

Caso 2: $k = B_0^j$

The initial loss-given-default of a core-bank is:

$$\frac{(1-\gamma)c_1\left[1-\frac{r}{R}\frac{c_2}{c_1}\right]}{1+(w_H-\gamma)\sum\limits_{i\in B^j\cup B^{-j}\setminus\{B_0^j\}}\frac{d_i}{d_{B_0^j}}}$$

Supposing $d_{B_0^j} \geq d_i$ $\forall i \in B^j \cup B^{-j}$, then the loss-give-default increases when the size heterogeneity gets higher.

i) Peripheral Counterpart

The exposure of a counterpart i to the central agent is given by $(w_H - \gamma)d_i$. Assuming $d_i = d$ $\forall i \in B^j \setminus \{B_0^j\}$ and $j \in \{A, B\}$, the inequality below might hold in case of contagion.

$$(w_H-\gamma)\frac{\left[1-\frac{r}{R}\frac{c_2}{c_1}\right]}{1+(w_H-\gamma)\left(1+2n\frac{d}{d_{B_0^j}}\right)} > \frac{r}{R}\left[\frac{c_2}{c_1}-1\right]$$

As a consequence, these counterparts suffer contagion more easily when size heterogeneity increases.

ii) Central Counterpart

In case of contagion of a central counterpart, the inequality below must hold.

$$\left(1+\sum_{i\in B^{-j}\setminus\{B_0^{-j}\}}\frac{d_i}{d_{B_0^{-j}}}\right)(w_H-\gamma)\frac{\left[1-\frac{r}{R}\frac{c_2}{c_1}\right]}{1+(w_H-\gamma)\sum\limits_{i\in B^j\cup B^{-j}\setminus\{B_0^j\}}\frac{d_i}{d_{B_0^j}}} > \frac{r}{R}\left[\frac{c_2}{c_1}-1\right]$$

Analysing the inequality above, it is clear that an higher size heterogeneity has two different effects on it. Suppose that $d = d_i$ $\forall i \in B^j \setminus \{B_0^j\}$, where $j \in \{A, B\}$. Furthermore, define $x \equiv \frac{d}{d_{B_0^j}}$, then:

$$\frac{(1+nx)(w_H-\gamma)}{1+w_H-\gamma+2n(w_H-\gamma)x} > \frac{\frac{r}{R}\left[\frac{c_2}{c_1}-1\right]}{\left[1-\frac{r}{R}\frac{c_2}{c_1}\right]}$$

Note that the derivative of the left side of the inequality (f) with respect to x is given below and, as a consequence, the result is proven.

$$\frac{\partial f}{\partial x} = \frac{n(w_H - \gamma)[1 - (w_H - \gamma)]}{[1 + w_H - \gamma + 2n(w_H - \gamma)x]^2} > 0$$

$\blacksquare$

9) Proof of Lemma 4

Suppose that $d_i = d \; \forall i \in B^j$ and $j \in \{A, B\}$, then the loss-given-default per unit of deposit is given by:

$$c_1 - q_{B_0^j} = \frac{(1-\gamma) d_{B_0^j} c_1 \left[1 - \frac{r}{R}\frac{c_2}{c_1}\right]}{d_{B_0^j} + (w_H - \gamma)(d_{B_0^{-j}} + 2nd)}$$

Thus,

$$\frac{\partial (c_1 - q_{B_0^j})}{\partial n} = -\frac{2d(w_H - \gamma)(1-\gamma) d_{B_0^j} c_1 \left[1 - \frac{r}{R}\frac{c_2}{c_1}\right]}{[d_{B_0^j} + (w_H - \gamma)(d_{B_0^{-j}} + 2nd)]^2} < 0$$

Since $\frac{R}{r} > \frac{c_2}{c_1}$, the derivative above is negative. I still need to verify the relation between the total loss induced to the central counterpart and n. The loss-given-default in that case is:

$$LGD_{B_0^j B_0^{-j}} = \frac{(w_H - \gamma)(d_{B_0^{-j}} + nd)(1-\gamma) c_1 \left[1 - \frac{c_2}{c_1}\frac{r}{R}\right]}{d_{B_0^j} + (w_H - \gamma)(d_{B_0^{-j}} + 2nd)} \tag{14}$$

Then,

$$\frac{\partial LGD_{B_0^j B_0^{-j}}}{\partial n} = \frac{d_{B_0^j}(1-\gamma)(1 - w_H + \gamma)(w_H - \gamma) d c_1 \left[1 - \frac{c_2}{c_1}\frac{r}{R}\right]}{[d_{B_0^j} + (w_H - \gamma)(d_{B_0^{-j}} + 2nd)]^2} > 0$$

$\blacksquare$

10) Proof of Lemma 5

First, I analyse the effects of a shock $\epsilon > \underline{\epsilon}$ in a core-bank. Suppose $d_i = d \ \forall i \in B^j \setminus \{B_0^j\}$ and $j \in \{A, B\}$, then:

$$q_{B_0^j} = \frac{\gamma d_{B_0^j} c_1 + (1-\gamma) d_{B_0^j} c_2 \frac{r}{R} + (w_H - \gamma) c_1 (d_{B_0^j} + 2nd)}{d_{B_0^j} + (w_H - \gamma)(2nd + d_{B_0^{-j}})}$$

Clearly, $\lim_{n\to\infty} q_{B_0^j} = c_1$. Thus, $\forall i \in B^j \setminus \{B_0^j\}$:

$$\lim_{n\to\infty} LGD_{B_0^j i} = \lim_{n\to\infty} (w_H - \gamma) d_i (c_1 - q_{B_0^j}) = (w_H - \gamma) d_i (c_1 - \lim_{n\to\infty} q_{B_0^j}) = 0$$

It means that peripheral banks of region j do not suffer contagion initially. However, I still need to analyse what happens to the central counterpart. If this institution does not go bankrupt, it is possible to say that the shock in a central agent is not able to cause a systemic collapse. The central counterpart does not need to liquidate all its assets when the inequality above is satisfied.

$$\lim_{n\to\infty} LGD_{B_0^j B_0^{-j}} < \lim_{n\to\infty} b_{B_0^{-j}}(\gamma)$$

Using (14) and knowing that the liquidity buffer does not depend on n, the inequality above is equivalent to:

$$\frac{(1-\gamma) d_{B_0^j} c_1}{2} \left[1 - \frac{r}{R}\frac{c_2}{c_1}\right] < d_{B_0^j} (1-\gamma) c_1 \frac{r}{R} \left[\frac{c_2}{c_1} - 1\right]$$

Then, the core-bank does not become insolvent when the inequality below holds.

$$\left[\frac{R}{r} - \frac{c_2}{c_1}\right] < 2\left[\frac{c_2}{c_1} - 1\right]$$

Since the contagion occurrence depends on parameters value, it is necessary to analyse the worst case. In other words, I should answer the following question: does the system collapse when both core-banks go bankrupt? In equilibrium, the payment per unit of deposit held in a core-bank is:

$$q_{B_0^j} = \frac{d_{B_0^j}\gamma c_1 + d_{B_0^j}(1-\gamma)c_2\frac{r}{R} + (w_H-\gamma)[2ndc_1 + (nd + d_{B_0^j})q_{B_0^{-j}}]}{d_{B_0^j} + (w_H-\gamma)(2nd + d_{B_0^{-j}})}$$

Once $q_{B_0^{-j}}$ is similar to $q_{B_0^j}$, I can write this variables as a function of parameters.

$$q_{B_0^j} = \frac{\left[d_{B_0^j}\gamma c_1 + d_{B_0^j}(1-\gamma)c_2\frac{r}{R}\right][d_{B_0^j} + (w_H-\gamma)(3nd + 2d_{B_0^j})]}{d^2_{B_0^j} + 2(w_H-\gamma)(2nd + d_{B_0^j}) + (w_H-\gamma)^2(3n^2d^2 + 2ndd_{B_0^j})} +$$

$$\frac{(w_H-\gamma)c_1nd[d_{B_0^j} + (w_H-\gamma)(2nd + d_{B_0^j}) + (w_H-\gamma)(nd + d_{B_0^j})]]}{d^2_{B_0^j} + 2(w_H-\gamma)(2nd + d_{B_0^j}) + (w_H-\gamma)^2(3n^2d^2 + 2ndd_{B_0^j})}$$

Therefore, it follows that:

$$\lim_{n\to\infty} q_{I_j} = c_1$$

It can be concluded that the loss-given-default induced by core-banks to peripheral institutions tends to zero, even if both central agents go bankrupt. The same is verified when a peripheral institution $k \in B_j$ is hit by a shock $\epsilon > \underline{\epsilon}$ and $d_{I_j} < d^M_{I_j}$.

■

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