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Implicit inflation and risk premiums for the Brazilian securities market

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Implicit inflation and risk premiums for the Brazilian securities market

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Abstract

The *breakeven inflation* (BEI), differential between nominal and real yields of bonds of the same maturity, is often used as a predictor of future inflation. The model presented here makes a decomposition of this interest rate differential in risk premiums and implied inflation using a parametric model based on no-arbitrage conditions. This model jointly estimates the two curves using a model of 4 factors of the Nelson-Siegel family. The results obtained have better forecasts on average squared error than the Focus Bulletin forecasts. The estimations of *breakeven inflation* and implied inflation of the model are shown unbiased estimators of future inflation for short horizons and carry some information for long horizons. The results also indicate that there are gains in the imposition of no-arbitrage.
Keywords: Inflation, Risk Premium and Bond Markets

1 Introduction

For proper management of a monetary policy, Central Banks should be interested in the inflation expectations of consumers and entrepreneurs. These agents' expectations are important for determining future inflation. In general, the data of future inflation comes from two main sources: the research expectations and the information contained in assets in the financial market. Data from surveys has a lower frequency, while the data of the financial assets market has a higher frequency. Another limitation of surveys is that we only have market expectations for horizons for 1 year, which means we do not have access to long-term inflation. So, the Central Bank can not verify if the market expectations of long-term inflation are aligned with the goals of the monetary authority.

Financial market prices can be accessed daily or even intraday form, and can generate information to a wider range of horizons. In Brazil we can estimate the expected inflation for up to five years in advnace. Moreover, as ponders Val, Barbedo & Maia (2011) surveys reflect the opinion of financial institutions, but not the bets they make

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on the market. Thus, when we analyze the information from financial market, we have daily updated information with different horizons, which reflects future expectations of the price that agents believe.

Due to these problems with expectations taken from surveys, different ways of extracting implicit expectations in financial assets have been proposed. A key work in this literature was Svensson & Soderling (1997), which discusses how to extract expectations through changes in asset prices. In our case, we use interest rate differentials between nominal bonds, which pay a fixed amount of interest, and real bonds, which are indexed to inflation plus a nominal interest rate. Nominal bonds have embedded in them the so-called implicit inflation. In this paper, to estimate expected inflation, we use Treasury bills (or LTN), which is a nominal title, and the National Treasury Notes Type B (or NTN-B), which are indexed to the IPCA titles (Consumer Index Price). We use NTN-B because the inflation that serves as an anchor for the decision making of policymakers to meet the inflation targets, besides being the title of greatest liquidity.

The interest rate differential between real and nominal bonds of the same maturity is known as breakeven inflation (BEI). The BEI is an indicator of expected inflation, however, it is an imperfect indicator because it includes inflation risk premiums. The bigggest problem is that the estimation of the risk premium in general depends on the specification of a parametric model, which defines the market risk premium for the uncertainty associated with future inflation.

Due that, the major contribution of this paper is that using a parametric model with no-arbitrage conditions, thus we can decompose de BEI rate in expected inflation and risk premium. Some articles, for Brazilian bond markets, use similar parametric models (eg Caldeira & Furlani (2014)) to extract the implicit inflations, but none of them imposes these restrictions. Thus, our work tries to get a more accurate measure of the expected inflation on financial markets.

Several researches were completed using government bonds to extract expected inflation for different markets (eg Woodward (1990) and Deacon & Derry (1994) for the British market, Shen & Corning (2001) for the American market, Alonso, Blanco & Rio (2001) for the French market). Christensen, Dion & Reid (2004) found a consistently higher than market expectations for the Canadian market. Therefore, concluding that market research is a consistent estimate of inflation expectations and the BEI rate is a biased estimate by the presence of risk premiums. Scholtes (2002) concludes that for the English case, the BEI rate has a better empirical performance than the surveys about inflation expectations.

The decomposition of the BEI rate in inflation risk premium and inflation expectations using a parametric model for the risk premium can be done through an affine free arbitrage model, which is most commonly used to represent the term structure of interest rates in finance. These models are typically specified using a set of latent factors that capture the movements in the yield curves in time. Duffie & Kan (1996) say that this model is popular because the yield curves are a linear function of the latent factors and loads, which is an advantage because of the existence of analytical solutions and the fact that this linear structure can be estimated using the Kalman filter. However, the canonical form of this model suffers from two problems. The first is that the maximum likelihood estimation of this class of model is subject to a problem of local maxima. Thus, we can obtain estimates with different economic implications. Moreover, it has a poor empirical performance to fit the observed and projected curves, as discussed in Duffee (2002).

Another line of research on parallel of modeling of the term structure is based on statistical models. These models are based on factorial representations without any direct economic interpretation as decomposition and principal component models based on the parameterization of Nelson & Siegel (1987), a dynamic formulation based on a structure of latent factors was proposed by Diebold & Li (2006). These models have a better fit of the curves and more accurate forecasts than the similar models, and so, were extensively adopted in the modeling of yield curves, as discussed in Diebold & Rudebusch (2011).

However, the absence of a complete economic interpretation, the lack of conditions for which there is no arbitrage and explicit ways to market premiums for risk preclude its use in applications requiring a structural interpretation of the results. A version of the Nelson-Siegel dynamic models with the imposition of no-arbitrage conditions in a formulation was proposed in order Christensen, Diebold & Rudebusch (2011), obtaining a model with empirical power and consistent with no-arbitrage. This model can decompose the BEI rate in implied inflation rate and risk premium. The authors conclude that their estimates are close to the financial market surveys and risk premiums vary around zero. Moreover, they suggest that the long-term expectations are well anchored. They also say that the use of this approach has the advantage of being easily updated and reestimated the Kalman filter, generating the advantage of having high frequency results in what is desirable for policy makers and financial market professionals.

For the Brazilian market, we can cite the work of Val, Barbedo & Maia (2011) which applies several different methodologies to estimate implicit inflation and compare it to the estimates of the BEI rate. The authors find a prize of relatively low inflation risk for the Brazilian market. And say that the surveys embody a prize greater risk than securities traded. Another important point is that the market for indexed bonds just happened to have an important role since 2006, with increases in liquidity and trading volume.

In the work of Vicente & Guillen (2013), the authors test whether breakeven inflation is a good measure of future inflation. With data from Brazilian government bonds, the authors perform two tests, using univariate regression methods. They perform a test to see if this breakeven inflation is an unbiased estimator of future inflation. The authors find evidence that breakeven inflation is an unbiased estimator for short horizons (3 and 6 months), longer for medium-term horizon(12 and 18 months). This variable has little explanatory power and for longer horizons (24 and 30 months) find one counterintuitive result that there is a negative relationship between future inflation rates and the BEI.

Caldeira & Furlani (2014) employ a Svensson (1994) model with four latent for nominal bonds and four factors for the real titles. From this model to find the BEI rate differential between bond yields. Then adopt a similar test used by Vicente & Guillen (2013) but beyond doing regressions also use a state-space formulation to perform the test. Thus, the authors find that the breakeven inflation is an unbiased estimator for inflation 3 months in advnace and it provides information only for shorter horizons. Also, compare the model predictions with those of a VAR model and market expectations of the Top 5 Focus Bulletin. The BEI rates show themselves superior to VAR models proposed by the authors, but worse than the Top 5.

The major contribution of the work of Christensen, Diebold & Rudebusch (2011) is to compare the results of the interest rate differential curves nominal and real interest with a Nelson-Siegel model with no-arbitrage restrictions. Through this model we can then decompose the BEI in expected inflation and risk premium. Here, we replicate this approach in order to compare models among themselves and with the predictions of the Focus Bulletin as a benchmark for the Brazilian market.

The paper is organized as follows: in the following section we describe the model and how we can decompose the interest rate differentials of indexed and nominal bonds in expected inflation and risk premium. Later, we will present the methodology used, the choice of specification and the coefficients found in Section 3. Section 4 presents the main results, the fit of the model and test whether our measure of implied inflation is useful for predicting future inflation, comparing the model results with the Focus Bulletin forecasts. To finalize, the Section 5 presents the conclusions and the most relevant results of the work.

2 Model

This section describes one Nelson-Siegel Arbitrage-Free Model in daily data from the BM&F/BOVESPA for LTN and NTN-B zero-coupon bonds. With this model we can decompose the interest rate differential between expected inflation and a risk premium. Below, we present a theoretical discussion of how to decompose the interest rate differentials in implicit inflation and risk premiums. Additionally, we present the empirical approach used in this work.

2.1 Theoretical Discussion

Proceeding with the approaches of Cochrane (2005) and Christensen, Lopez & Rudebusch (2010) we will decompose the spread between real and nominal bonds in inflation implied and risk premium using a formulation in continuous time. Defining M_t^R

e M_t^N as stochastic discount factors for the real and nominal, respectively. By no-arbitrage conditions we have:

$$P_t^i M_t^i = E_t^P \left[P_{t+\tau}^i M_{t+\tau}^i \right] \tag{1}$$

with $i = R, N$. This equation represents the decision making of the agent forgoing present consumption to invest and consume in the future. Equality in 1 is derived from the first order condition, for details see Cochrane (2005). So, we can normalize the price $P_{t+\tau}^i = 1$, i.e. an unitatry payoff in period τ and rewrite prices as:

$$P(\tau)_t^i = E_t^P \left[\frac{M_{t+\tau}^i}{M_t^i} \right] \tag{2}$$

In addition, the no-arbitrage condition requires that there be consistency between the prices of real and nominal bonds. Thus, we can define Q_t as the general price level, which is nothing more than the relationship between the stochastic discount factors:

$$Q_t = \frac{M_t^R}{M_t^N} \tag{3}$$

Under the hypothesis that the stochastic discount factors has the following dynamics:

$$\frac{dM_t^i}{M_t^i} = -r_t^i dt - \Gamma_t dW_t^P \tag{4}$$

ie the drift part of the process is determined by the instantaneous interest rate, which varies for each of the types of securities, and a part that depends on the diffusion Γ_t, which is the market price of risk is constant between the types of securities and the shocks of Brownian motion. From 3, then we can find by Itô's Lemma, the dynamics of the price level:

$$dQ_t = (r_t^R - r_t^N) Q_t dt \tag{5}$$

therefore, in the absence of arbitrage the instantaneous rate of growth of prices is determined by the instantaneous differential interest rates between real and nominal bonds, so the Fisher equation holds for the instantaneous rate and there is no risk premium. Again using Ito's Lemma, we can see that:

$$dln(Q_t) = \frac{1}{Q_t} dQ_t - \frac{1}{2} \frac{1}{Q_t} dQ_t^2 = \frac{1}{Q_t} dQ_t = (r_t^R - r_t^N) dt \tag{6}$$

integrating both sides and taking the exponential we have:

$$Q_{t+\tau} = Q_t e^{\int_t^{t+\tau}(r_t^R - r_t^N)dt} \tag{7}$$

With this we can write the price of the nominal title due to price level and the real price, so:

$$P(\tau)_t^N = E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^P\left[\frac{Q_t}{Q_{t+\tau}}\right] \times \left(1 + \frac{cov(\frac{M_{t+\tau}^R}{M_t^R}, \frac{Q_t}{Q_{t+\tau}})}{E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^P\left[\frac{Q_t}{Q_{t+\tau}}\right]}\right) \tag{8}$$

converting the asset price in yield:

$$P(\tau)_t^N = P_t^N e^{y_t^N} \rightarrow y_t^N = -\frac{1}{\tau} ln\left[P_t^N / P(\tau)_t^N\right] \tag{9}$$

we can rewrite 8 as:

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau) \tag{10}$$

being $y_t^N(\tau)$ the nominal bonds yield to maturityτ, $y_t^R(\tau)$, the real bond yield for the same maturity, $\pi_t^e(\tau)$ implicit inflation bond market in period t to period $t+\tau$ and $\phi_t(\tau)$ the risk premium to hold nominal title until $t+\tau$. We can write the implicit inflation as:

$$\pi_t^e(\tau) = -\frac{1}{\tau} ln\left[Q_t / Q_{t+\tau}\right] = -\frac{1}{\tau} ln E_t^P\left[e^{-\int_t^{t+\tau}(r_t^R - r_t^N)dt}\right] \tag{11}$$

and the risk premium as:

$$\phi_t(\tau) = -\frac{1}{\tau} ln\left(1 + \frac{cov(\frac{M_{t+\tau}^R}{M_t^R}, \frac{Q_t}{Q_{t+\tau}})}{E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^P\left[\frac{Q_t}{Q_{t+\tau}}\right]}\right) \tag{12}$$

therefore, the risk premium on keeping a nominal title can be positive or negative. The risk premium will be positive only if the $cov(\frac{M_{t+\tau}^R}{M_t^R}, \frac{Q_t}{Q_{t+\tau}}) < 0$, i.e., an increase in the price level, which is a moment of loss of purchasing power, has a positive effect on the price of real titles.

As defined in the first part of this work the BEI (breakeven inflation) to $t+\tau$ is the interest rate differential between real and nominal bonds to maturity τ. As we saw in equation 10, we can decompose the BEI in implicit inflation and risk premium:

$$BEI(\tau)_t = y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau) \tag{13}$$

2.2 Empirical Approach

To know what the best specification for the joint model for real and nominal bonds is, we need to determine the number of latent factors in the model. In general, the models for term structure of interest rates use three factors: the level, slope and curvature. These factors are generally sufficient to explain the variation cross-section of titles. The first term is the level that determines the long-term factors. The second factor can be considered the slope factor or short term or the spread while the third factor is considered the curvature factor or medium term.

To test how many factors it is necessary, we use the principal component analysis, analyzing data from January 2006 to October 2013 for nominal bonds with a maturity of 1, 3, 6, 12, 15, 18, 21, 24, 30, 36 , 42, 48 and 60 months and for indexed bonds for the same maturities. We use this period because IPCA indexed titles seem to have little liquidity for periods before this. Table 1 shows the principal components analysis in order to see how many components are sufficient to explain the curves, four factors explain more than 99.2% of the curves. Thus, we propose a model with: two factors of levels, one for each type of security, a slope factor and one factor common curvature.

Christensen, Lopez & Rudebusch (2010) showed that the Arbitrage-Free with Nelson-Siegel factors model has a good degree of adjustment to the term structure of interest rates and generates good predictions out of sample. To capture possible differences in scale between the slopes of the nominal and real bonds, also estimate the α, which was not done in previous works.

The Q dynamic of the Nelson-Siegel factors for nominal bonds is given by the following stochastic differential equations:

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\lambda & \lambda & 0 \\ 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L_{t-1}^N - \theta_1^{P,J} \\ S_{t-1} - \theta_2^{P,J} \\ C_{t-1} - \theta_3^{P,J} \\ L_{t-1}^R - \theta_4^{P,J} \end{pmatrix} dt + \Sigma^j \begin{pmatrix} DW^{L_t^N,P} \\ DW^{S,P} \\ DW^{C,P} \\ DW^{L_t^R,P} \end{pmatrix} \tag{14}$$

and the P dynamics is given by:

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \end{pmatrix} = \begin{pmatrix} K_{11}^{P,J} & K_{12}^{P,J} & K_{13}^{P,J} & K_{14}^{P,J} \\ K_{21}^{P,J} & K_{22}^{P,J} & K_{23}^{P,J} & K_{24}^{P,J} \\ K_{31}^{P,J} & K_{32}^{P,J} & K_{33}^{P,J} & K_{34}^{P,J} \\ K_{41}^{P,J} & K_{42}^{P,J} & K_{43}^{P,J} & K_{44}^{P,J} \end{pmatrix} \begin{pmatrix} L_{t-1}^N - \theta_1^{P,J} \\ S_{t-1} - \theta_2^{P,J} \\ C_{t-1} - \theta_3^{P,J} \\ L_{t-1}^R - \theta_4^{P,J} \end{pmatrix} dt + \Sigma^j \begin{pmatrix} DW^{L_t^N,P} \\ DW^{S,P} \\ DW^{C,P} \\ DW^{L_t^R,P} \end{pmatrix} \tag{15}$$

with diagonal Σ^j. As we saw in the previous section, the momentum P is very important

Table 1 – Principal component analysis of indexed and nominal bonds

Maturity						
Real	1º PC	2º PC	3º PC	4º PC	5º PC	6º PC
1 Month	0.134553	0.467604	0.014705	0.78545	-0.27805	0.258406
3 Month	0.167591	0.357665	-0.04919	0.114176	0.344071	-0.6968
6 Month	0.179627	0.265103	-0.06309	-0.05793	0.038507	-0.29027
9 Month	0.184117	0.190391	-0.07081	-0.14773	-0.0671	-0.12424
12 Month	0.185925	0.142082	-0.08897	-0.17352	-0.13651	-0.02966
15 Month	0.187	0.102497	-0.11464	-0.16004	-0.16635	-0.01608
18 Month	0.187608	0.069742	-0.1429	-0.13853	-0.15875	0.002999
21 Month	0.187898	0.044778	-0.16131	-0.12077	-0.13889	0.036049
24 Month	0.188104	0.024581	-0.17109	-0.10158	-0.11191	0.06148
26 Month	0.188119	0.014526	-0.17766	-0.09155	-0.09786	0.07519
30 Month	0.188029	0.002117	-0.19071	-0.07273	-0.06763	0.095541
36 Month	0.187785	-0.01158	-0.20881	-0.0501	-0.01124	0.109623
42 Month	0.187566	-0.01581	-0.21758	-0.03581	0.040968	0.136009
48 Month	0.187261	-0.01559	-0.22502	-0.02648	0.096531	0.159568
60 Month	0.185982	-0.0109	-0.25463	-0.02256	0.192267	0.217021
Nominal						
1 Month	0.17434	0.24443	0.298763	-0.13154	0.433621	0.344089
3 Month	0.178917	0.184721	0.32384	-0.12935	0.265464	0.226057
6 Month	0.18341	0.087963	0.325201	-0.11281	0.041807	0.086815
9 Month	0.185225	0.013704	0.303282	-0.08272	-0.0904	0.003123
12 Month	0.18595	-0.04622	0.267105	-0.04997	-0.16666	-0.05446
15 Month	0.186054	-0.09489	0.225593	-0.01842	-0.19623	-0.08444
18 Month	0.185835	-0.13181	0.18619	0.012417	-0.19224	-0.08931
21 Month	0.18548	-0.16085	0.14918	0.040502	-0.16129	-0.08079
24 Month	0.185047	-0.18271	0.116719	0.06492	-0.1155	-0.07936
26 Month	0.184668	-0.19485	0.099642	0.081025	-0.08334	-0.07678
30 Month	0.183836	-0.21435	0.064917	0.114015	-0.02108	-0.06584
36 Month	0.182727	-0.2304	0.014199	0.154256	0.072478	-0.0617
42 Month	0.181714	-0.23768	-0.03153	0.179167	0.168997	-0.03893
48 Month	0.180514	-0.24503	-0.06206	0.196124	0.245654	-0.01913
60 Month	0.178209	-0.25433	-0.11583	0.223583	0.346699	0.008125
% explained	0.9216	0.964	0.981	0.9922	0.9961	0.9986

to decompose the BEI in implicit inflation and risk premium. The yield curves are:

$$y(\tau)^N = L_t^N + S_t((1-e^{-\lambda\tau})/\lambda\tau) + C_t((1-e^{-\lambda\tau})/\lambda\tau - e^{-\lambda\tau}) + \epsilon_{N,\tau} \tag{16}$$

$$y(\tau)^R = L_t^R + \alpha S_t((1-e^{-\lambda\tau})/\lambda\tau) + \alpha C_t((1-e^{-\lambda\tau})/\lambda\tau - e^{-\lambda\tau}) + \epsilon_{R,\tau} \tag{17}$$

being L_t^N e L_t^R the factors of level of nominal and real bonds, respectively, S_t the slope factor and C_t, the curvature factor and α a term adjustment for the real bonds.

To facilitate the estimation, we will discretize this process as Mouabbi (2013). Being $X_t = (L_t^N, S_t, C_t, L_t^R)$, we can rewrite equation 15 as:

$$X_T = [I - e^{K^P(T-t)}]\theta^P + e^{K^P(T-t)}X_t + \eta_t \tag{18}$$

thus the transition equation can be estimated in discrete time as a VAR (1). This estimation procedure has been used in Christensen, Lopez & Rudebusch (2012) by quasi-maximum likelihood using the Kalman filter. This transformation allows us to estimate and recover the parameters of the P dynamic from equation 15. In the work of Christensen, Lopez & Rudebusch (2010) the authors show that for a Gaussian approach we can rewrite the equation:

$$E_t^P\left[e^{-\int_t^{t+\tau}(r_t^R-r_t^N)dt}\right] = exp\left[B^{\pi}(\tau)X_t + A^{\pi}(\tau)\right] \tag{19}$$

$B^{\pi}(\tau)$ and $A^{\pi}(\tau)$ being the solutions of differential equations solved by the Runge-Kutta fourth order. From this we can calculate the implied inflation and risk premiums [3] .

3 Estimation

The estimates made in this study used the methodology of the Kalman filter, following Christensen, Lopez & Rudebusch (2012). The measurement equations 16 and 17 can be rewritten as:

$$y_t = BX_t + \epsilon_t \tag{20}$$

in which ϵ_t represent errors of measurement equations that are independent and identically distributed assumption (iid) for each maturity. The equation of state of the factors is given by Equation 18, can be rewritten as:

$$X_t = \Theta + \Phi X_{t-1} + \eta_t \tag{21}$$

error structure given by:

$$\begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{matrix} R & 0 \\ 0 & Q \end{matrix}\right) \tag{22}$$

where $R = diag(\sigma_i^2)$ (4×4) and measurement equations have the same variance with $Q = diag(\sigma^2)$. Furthermore, the optimality of the Kalman filter needs that errors are orthogonal to the state variable: $E(X_t\eta_t) = 0$ e $E(X_t\epsilon_t) = 0$.

An estimation must be both flexible and parsimonious. Thus, it is important that the model is well specified. Additionally, to decompose the BEI rate it is very important that our dynamic P is correct. Therefore, we use several statistical tests with the information criterion to choose the best specification. The results of these tests are presented in the table 2. All information criterion (Akaike information criterion (AIC), Bayesian (BIC) and Hannan-Quinn) indicate that the best specification is the diagonal model.

Table 3 shows estimated values for the chosen model. In this table we present the estimates for the matrix ϕ parameters,θ, the Σ in addition to the decay factor of the measure (λ) equation and α for the actual titles . We can see that we estimate a decay factor 0.19, near the value of other work applied to Brazil. The parameter α was estimated to 0.97, the work of Christensen, Lopez & Rudebusch (2010) finds a value of 0.92,

[3] For more details see Christensen, Diebold & Rudebusch (2011).

Table 2 – Specification Tests

	Tests				
	logL	Parameters	AIC	BIC	Hannan-Quinn
Unrestricted	-32612.9	27	34.10648	34.18487	34.13533
1 Restriction	-32853.9	26	34.33927	34.41473	34.36704
2 Restriction	-32836.5	25	34.32009	34.39264	34.34679
3 Restriction	-32546.7	24	34.01634	34.08599	34.04197
4 Restriction	-32569	23	34.03865	34.1054	34.06321
5 Restriction	-32393.3	22	33.85406	33.9179	33.87755
6 Restriction	-32339	21	33.79632	33.85726	33.81875
7 Restriction	-32338.1	20	33.79432	33.85236	33.81568
8 Restriction	-32339.5	19	33.79479	33.84993	33.81508
9 Restriction	-32159.1	18	33.60533	33.65757	33.62456
10 Restriction	-32161.2	17	33.60646	33.65579	33.62461
11 Restriction	-31986.1	16	33.42252	33.46895	33.43961
Diagonal	-31972.2	15	**33.40701**	**33.45054**	**33.42303**

Note - We imposed the least significant parameter was equal to zero in each constraint.[4]

Table 3 – Estimated Parameters of the chosen model

	$\phi, 1$	$\phi, 2$	$\phi, 3$	$\phi, 4$	θ	α	λ
$\phi 1$,	0.995				-0.016	0.972	0.193
	(0.001184)				(0.009498)	(0.00000689)	(0.000000759)
$\phi 2$,		0.878			0.264		
		(0.002268)			(0.007128)		
$\phi 3$,			0.972		0.060		
			(0.003295)		(0.017272)		
$\phi 4$,				1.000	-0.007		
				(0.000119)	(0.00000968)		

for example.

4 Results

In this section we present, first, the values found for the latent factors, as well as the forecasts for the analyzed bonds. Later we will show the values found for the implicit inflation and risk premiums for the domestic market. And propose a test to see if the implicit inflation is a good predictor of the future inflation.

4.1 Latent factors and adjustment of bonds

Figures 1 and 2 show the level, slope and curvature factors estimates for real and nominal bonds for the period analyzed. The correlation between these two factors is the

[4] Model with 1 constraint: $\phi_{1,4} = 0$. Model with 2 constraints: $\phi_{1,4} = \phi_{4,1} = 0$. Model with 3 constraints: $\phi_{1,4} = \phi_{4,1} = \phi_{4,3} = 0$. Model with 4 constraints: $\phi_{1,4} = \phi_{4,1} = \phi_{4,3} = \phi_{3,4} = 0$. Model with 5 constraints: $\phi_{1,4} = \phi_{4,1} = \phi_{4,3} = \phi_{3,4} = \phi_{2,4} = 0$. Model with 6 constraints: $\phi_{1,4} = \phi_{4,1} = \phi_{4,3} = \phi_{3,4} = \phi_{2,4} = \phi_{3,1} = 0$. Model with 7 constraints: $\phi_{1,4} = \phi_{4,1} = \phi_{4,3} = \phi_{3,4} = \phi_{2,4} = \phi_{3,1} = \phi_{4,2} = 0$. Model with 8 constraints: $\phi_{1,4} = \phi_{4,1} = \phi_{4,3} = \phi_{3,4} = \phi_{2,4} = \phi_{3,1} = \phi_{4,2} = \phi_{1,3} = 0$. Model with 9 constraints: $\phi_{1,4} = \phi_{4,1} = \phi_{4,3} = \phi_{3,4} = \phi_{2,4} = \phi_{3,1} = \phi_{4,2} = \phi_{1,3} = \phi_{2,1} = 0$. Model with 10 constraints: $\phi_{1,4} = \phi_{4,1} = \phi_{4,3} = \phi_{3,4} = \phi_{2,4} = \phi_{3,1} = \phi_{4,2} = \phi_{1,3} = \phi_{2,1} = \phi_{1,2} = 0$. Model with 11 constraints: $\phi_{1,4} = \phi_{4,1} = \phi_{4,3} = \phi_{3,4} = \phi_{2,4} = \phi_{3,1} = \phi_{4,2} = \phi_{1,3} = \phi_{2,1} = \phi_{1,2} = \phi_{3,2} = 0$.

level of 91.71 %, and we note that both the real and the nominal level factor show an increase for 2008, the period of the Lehmann Brothers crisis. This shows that the crisis influenced both real and nominal bonds - probably by the macroeconomic channel. In work of Diebold, Rudebusch & Aruoba (2006) the authors indicate that inflation influences the level factor. This may explain the increase of this factor both for 2008 and for 2013 The curvature and slope factors are much less volatile than the level following the literature (e.g. Diebold & Li (2006)).

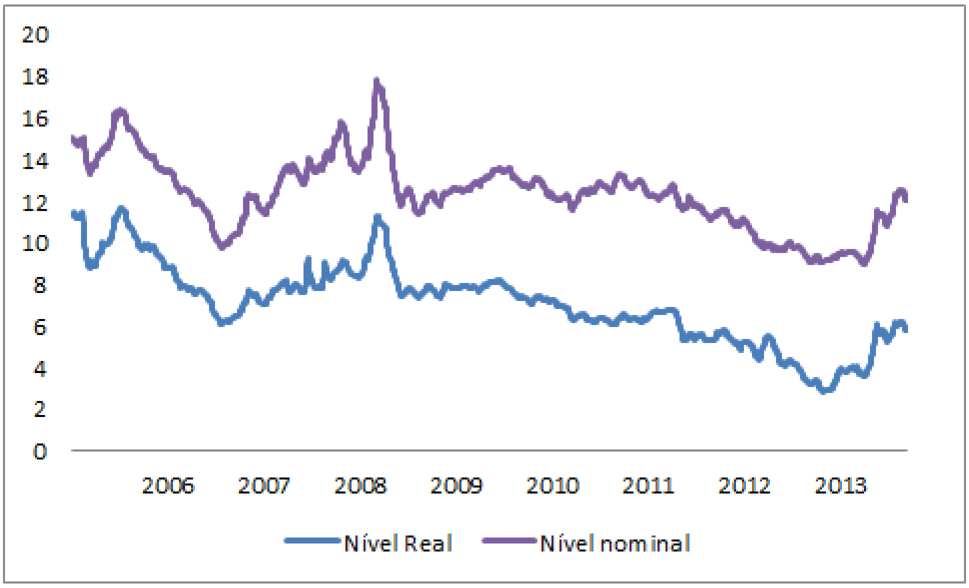

Figure 1 – Level Factors Estimated

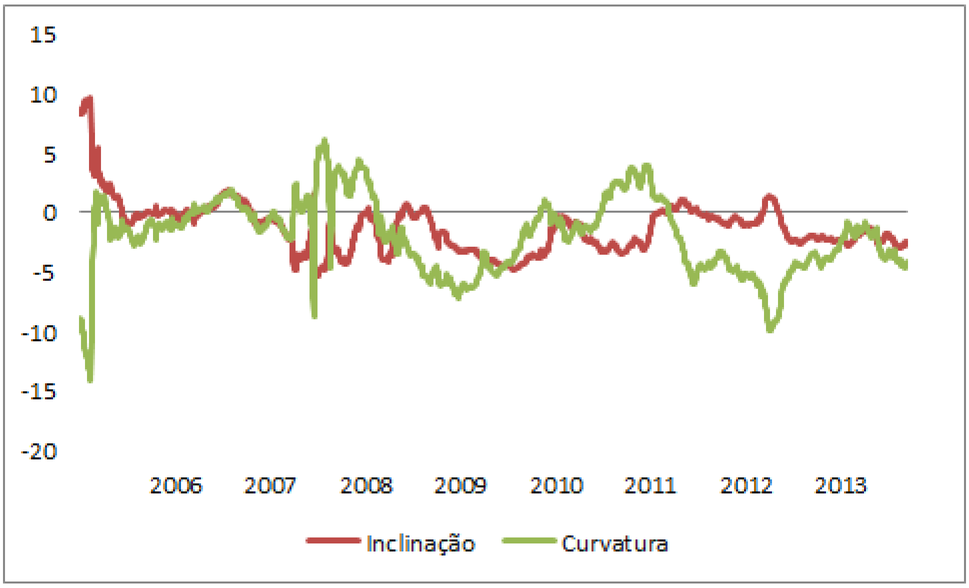

Figure 2 – Slope and curvature factors estimated

Table 4 shows measures of fit of the model for different maturity of the bonds in terms of Medium Error and the Root Mean Square Error. These measurements show that

Table 4 – Adjustment measures

	Mean Error	Root Mean Square Error
Real		
Maturity 1 Month	-0.095609	2.798753
Maturity 3 Months	0.05489	0.57199
Maturity 6 Months	-0.015383	0.195772
Maturity 9 Months	-0.007725	0.113417
Maturity 12 Months	0.001921	0.086818
Maturity 15 Months	-0.001644	0.07728
Maturity 18 Months	-0.00026	0.074582
Maturity 21 Months	-0.002758	0.077102
Maturity 24 Months	-0.007826	0.079358
Maturity 26 Months	-0.009574	0.081415
Maturity 30 Months	-0.007437	0.083792
Maturity 36 Months	0.01013	0.085904
Maturity 42 Months	0.036823	0.09254
Maturity 48 Months	0.063338	0.100567
Maturity 60 Months	0.128385	0.139267
Nominal		
Maturity 1 Month	0.042264	0.876392
Maturity 3 Months	0.02634	0.40592
Maturity 6 Months	0.028155	0.224233
Maturity 9 Months	0.046288	0.199712
Maturity 12 Months	0.047124	0.203884
Maturity 15 Months	0.028979	0.209178
Maturity 18 Months	0.009476	0.210486
Maturity 21 Months	-0.007772	0.204806
Maturity 24 Months	-0.023365	0.197345
Maturity 26 Months	-0.032977	0.19443
Maturity 30 Months	-0.040021	0.193084
Maturity 36 Months	-0.04179	0.196226
Maturity 42 Months	-0.049769	0.203946
Maturity 48 Months	-0.053981	0.211662
Maturity 60 Months	-0.049313	0.235099

the model generally provides a good fit, especially for real bonds, as can be seen by the smaller squared errors for these series.

4.2 Implicit Inflation and Risk Premiums

From the equation 19, we can calculate the implied inflation of our model. As discussed in section 2, the BEI rate can be decomposed into implicit inflation and risk premium. From equation 13 and taking the implicit inflation as described, we can calculate the risk premium for each maturity.

Using the same approach of Christensen, Lopez & Rudebusch (2010), we compared our results with market forecasts. In the present work we use as a benchmark the average forecasts of market expectations published in the Bulletin Focus. The model results are close to the Focus Bulletin forecasts over time. This may be the result of a more efficient bond market or the loss of credibility of BC in the conduct of monetary policy leading agents to report less conservative predictions.

We can see in Figures 3 and 4 that the implicit inflation model seems to be better than the Focus forecasts for both the horizon of 6 and 12 months. When we see that both the mean error and the mean squared error are smaller than those of the Focus forecasts,

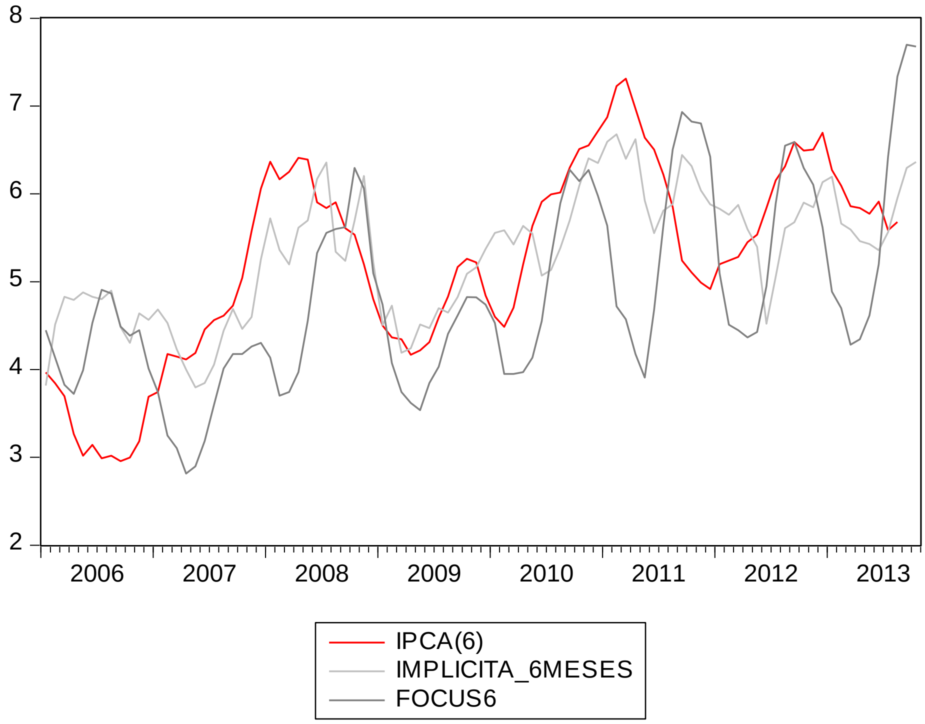

Figure 3 – Focus Bulletin, Expected Inflation Model and IPCA performed 6 Months in advnace

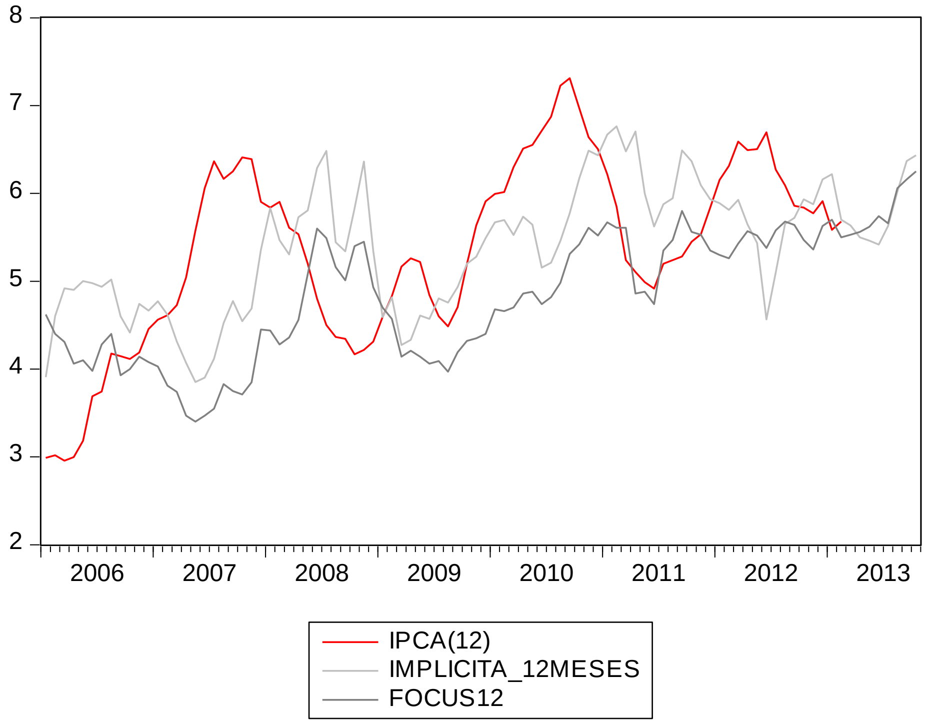

Figure 4 – Focus Bulletin, Expected Inflation Model and IPCA performed 12 Months in advnace

in Table 5. When comparing the BEI rates with the implicit inflation, we also see that the implied inflation seems to be a superior predictor.

Figure 5 presents the risk premium model for maturities of 6, 12, 24 and 60 months.

Table 5 – Fit of the model

	6 Month		12 Month	
	ME	**RMSE**	**ME**	**RMSE**
Implicita	-0.04912	0.592281	0.006084	1.127463
BEIR	0.043947	0.75811	0.195837	1.247082
FOCUS[5]	0.454405	1.471076	0.619568	1.456684

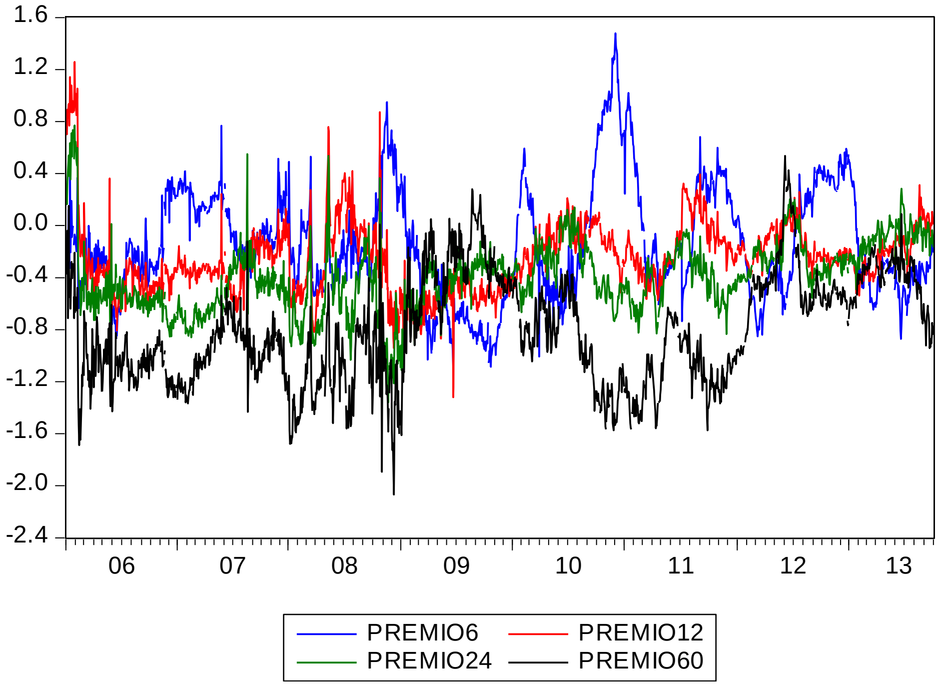

Figure 5 – Risk premiums for 6, 12, 24 and 60 months of maturity

We see that risk premiums are higher for shorter horizons. The dynamics of risk premiums seems to be the same between the bonds of up to 24 Months, with only a difference of level. As for longer horizons (60 Month) agents can anchor inflation expectations in the long terms, which may explain the different dynamics for these bonds. Another interesting point of this work is that risk premiums seem to be variants both in time and in maturity.

4.3 Implicit inflation as a predictor of future inflation

The work of Vicente & Guillen (2013) and Caldeira & Furlani (2014) propose a methodology to test the predictive power of the implicit inflation on future inflation. Defining $h(1)_t$ as the annual rate continuously compounded interest, we can define the cumulative inflation between period t and $t+\tau$ as $h(\tau)_t = \frac{1}{\tau}\sum_{j=t}^{j=t+\tau} h_j(1)$. We can test

[5] This measure is the mean market forecast, the median forecast of the market had worse results thus we use the mean.

whether the implied inflation has predictive power on inflation using a regression with the following functional form:

$$h(\tau)_t = c_0 + c_1\pi_t^e(\tau) + \epsilon_{t+\tau} \tag{23}$$

the equation 23 demonstrates that the term c_1 is significant, then the implied inflation provides some information about future inflation, and if $c_0 = 0$ and $c_1 = 1$ implicit inflation estimator is an unbiased future inflation. As Vicente & Guillen (2013) first estimate the equation 23 by Method of Ordinary Least Squares (OLS). Later, to avoid possible endogeneity problems we will make an estimation by Two-Stage Least Squares (TSLS) with the first lag of implied inflation as a tool and an estimation by Generalized Method of Moments (GMM) with the first three lags as instruments. The authors propose an estimation by TSLS and GMM for future inflation should be influenced by inflation expectations as well as expectations for inflation are contaminated performed, generating endogeneity. With this, we test the significance of c_1 and if $c_0 = 0$ e $c_1 = 1$ through a Wald tes. Another point worth noting is that Vicente & Guillen (2013) said their estimates are correct only under the assumption that risk premium are constant over time, which does not seem to be true for the Brazilian case.

Table 6 – OLS of the model's Implicit Inflation

	Maturity							
	3	6	9	12	24	36	48	60
c_0	1.15	1.10	0.83	0.43	0.10	0.65	-0.53	0.40
	0.00	0.00	0.00	0.07	0.60	0.00	0.00	0.03
c_1	-0.01	-0.01	0.01	0.03	0.05	0.02	0.09	0.04
	0.24	0.53	0.44	0.02	0.00	0.02	0.00	0.00
R^2	56.2%	50.9%	29.3%	8.1%	-0.4%	34.2%	22.7%	21.6%
F-test	0.30	0.82	0.62	0.04	0.00	0.06	0.00	0.00

The values below the coefficients are the P-value of coefficients

Table 7 – TSLS of the model's Implicit Inflation

	Maturity							
	3	6	9	12	24	36	48	60
c_0	1.35	1.18	0.84	0.37	0.16	0.84	-0.72	0.50
	0.00	0.00	0.00	0.18	0.52	0.00	0.00	0.02
c_1	-0.02	-0.01	0.01	0.03	0.05	0.01	0.10	0.03
	0.04	0.39	0.53	0.03	0.00	0.41	0.00	0.01
R^2	56.3%	50.1%	27.1%	5.8%	-1.7%	34.8%	29.3%	21.2%
F-test	0.08	0.68	0.71	0.06	0.00	0.42	0.00	0.01

The values below the coefficients are the P-value of coefficients

Tables 6, 7 and 8 have the tests using the estimates proposed by Vicente & Guillen (2013). We can see that, in general, the tests are robust and show the same results. As for the BEI rates, the results are not robust, indicating different results depending on which

Table 8 – GMM of the model's Implicit Inflation

	Maturity							
	3	6	9	12	24	36	48	60
c_0	1.20	1.05	0.76	0.27	-0.02	0.70	-0.70	0.40
	0.00	0.00	0.00	0.17	0.93	0.00	0.00	0.04
c_1	-0.01	0.00	0.01	0.04	0.06	0.02	0.10	0.04
	0.16	0.78	0.25	0.00	0.00	0.07	0.00	0.00
R^2	55.2%	49.3%	25.9%	4.2%	-2.9%	33.1%	32.8%	20.5%
F-test	0.02	0.96	0.45	0.00	0.00	0.16	0.00	0.00

The values below the coefficients are the P-value of coefficients

estimation method we are using. Vicente & Guillen (2013) point that the basic assumption of the proposed test for them is that risk premiums are constant in time, which for the Brazilian case doesn't seem to be true. Estimates of test for BEI rates are present in Annex A.

The implied inflation seems to be an unbiased estimator for up to nine months, showing greater results than those found in the works of Vicente & Guillen (2013) and Caldeira & Furlani (2014). The results also corroborate that for the medium term (24-48 months) the titles do not seem to provide good information. The results for implicit inflation for 36 months indicate that it is an unbiased estimator of realized inflation. For the long term (60 months) implicit inflation seems to provide information explaining more than 20% of the variation in inflation. The results regarding the superiority of one of the measurements indicate that there are gains when we impose no-arbitrage restrictions on forecasts of inflation, as well as providing information on future inflation and should generate better outcomes for which the risk premium is too large .

5 Conclusions

The aim of this study was to use the nominal and real bond markets to estimate inflation from them. For this we use nominal bonds and notes National Treasury of type B. We use the methodology proposed by latent factors Nelson & Siegel (1987) in a dynamic manner, based on the work of Christensen, Lopez & Rudebusch (2010) estimate a free arbitrage model that uses four factors: 2 different for level 2 and factor common to the slope and curvature factors. With several criteria specification, we chose the most parsimonious model This model four factors, in its diagonal form, has a good fit for values of nominal and indexed bonds. The results presented here for both the implied inflation rates and the BEI rate show better forecasts for 6 and 12 months in advnace compared to average of market forecasts presented in the Focus Bulletin.

The results of the proposed test by Vicente & Guillen (2013) show that imposing no-arbitrage restrictions increase the predictive power of the model. Furthermore, the test seems to be inconsistent for BEI rates as it has for the hypothesis that the risk premiums are not variant in time (our model presented awards of time-varying risk). This test indicates, for shorter horizons the forecasts of the implicit inflation are unbiased estimators of inflation future for up to nine months horizons. As for medium-term and long-term horizons, the results are inconclusive, as already shown in other studies applied to the Brazilian market. This paper closes a gap in the literature of the use of government bonds for forecasting inflation in Brazil with the decomposition of BEI rate implicit in inflation and risk premiums with no-arbitrage restrictions for Brazilian market

Bibliography

ALONSO, F.; BLANCO, R.; RIO, A. del. *Estimating inflation expectations using French government inflation-indexed bonds*. [S.l.]: Banco de Espana, Servicio de Estudios, 2001.

CALDEIRA, J. F.; FURLANI, L. Inflação implícita eo prêmio pelo risco: Uma alternativa aos modelos var na previsão para o ipca. *Estudos Econômicos (São Paulo)*, v. 43, n. 4, 2014.

CHRISTENSEN, I.; DION, F.; REID, C. *Real Return Bonds, Inflation Expectations, and the Break-Even Inflation Rate*. [S.l.]: Bank of Canada, 2004.

CHRISTENSEN, J. H.; DIEBOLD, F. X.; RUDEBUSCH, G. D. The affine arbitrage-free class of Nelson-Siegel term structure models. *Journal of Econometrics*, v. 164, n. 1, p. 4 – 20, 2011. Annals Issue on Forecasting.

CHRISTENSEN, J. H.; LOPEZ, J. A.; RUDEBUSCH, G. D. Inflation expectations and risk premiums in an arbitrage-free model of nominal and real bond yields. *Journal of Money, Credit and Banking*, Wiley Online Library, v. 42, n. s1, p. 143–178, 2010.

CHRISTENSEN, J. H. E.; LOPEZ, J. a.; RUDEBUSCH, G. D. Can Spanned Term Structure Factors Drive Stochastic Yield Volatility? *SSRN Electronic Journal*, 2012. ISSN 1556-5068. Disponível em: <http://www.ssrn.com/abstract=2023715>.

COCHRANE, J. H. *Asset pricing*. [S.l.]: Princeton university press Princeton, 2005.

DEACON, M.; DERRY, A. Estimating market interest rate and inflation expectations from the prices of uk government bonds. *Bank of England Quarterly Bulletin*, v. 34, n. 3, p. 232–240, 1994.

DIEBOLD, F. X.; LI, C. Forecasting the term structure of government bond yields. *Journal of econometrics*, Elsevier, v. 130, n. 2, p. 337–364, 2006.

DIEBOLD, F. X.; RUDEBUSCH, G. D. Yield Curve Modeling and Forecasting The Dynamic Nelson-Siegel Approach. 2011.

DIEBOLD, F. X.; RUDEBUSCH, G. D.; ARUOBA, S. B. The macroeconomy and the yield curve: a dynamic latent factor approach. *Journal of econometrics*, Elsevier, v. 131, n. 1, p. 309–338, 2006.

DUFFEE, G. R. Term premia and interest rate forecasts in affine models. *The Journal of Finance*, Wiley Online Library, v. 57, n. 1, p. 405–443, 2002.

DUFFIE, D.; KAN, R. A yield-factor model of interest rates. *Mathematical finance*, Wiley Online Library, v. 6, n. 4, p. 379–406, 1996.

MOUABBI, S. An arbitrage-free nelson-siegel term structure model with stochastic volatility for determination of risk premia. 2013. Job Market Paper - Queen Mary, University of London - November, 2013.

NELSON, C. R.; SIEGEL, A. F. Parsimonious modeling of yield curves. *Journal of business*, JSTOR, p. 473–489, 1987.

SCHOLTES, C. On market-based measures of inflation expectations. *Bank of England Quarterly Bulletin, Spring*, 2002.

SHEN, P.; CORNING, J. Can tips help identify long-term inflation expectations? *Economic Review-Federal Reserve Bank of Kansas City*, Federal Reserve Bank of Kansas City, v. 86, n. 4, p. 61–87, 2001.

SVENSSON, L.; SODERLING, P. New techniques to extract market expectations from financial instruments. *Journal of Monetary Economics,*, v. 40, p. 383–429, 1997.

SVENSSON, L. E. *Estimating and interpreting forward interest rates: Sweden 1992-1994*. [S.l.], 1994.

VAL, F. de F.; BARBEDO, C. H. da S.; MAIA, M. V. Expectativas inflacionárias e inflação implícita: será que pesquisas de mercado fornecem medidas precisas? *BBR-Brazilian Business Review*, FUCAPE Business School, n. 3, p. 88–107, 2011.

VICENTE, J. V. M.; GUILLEN, O. T. d. C. Do inflation-linked bonds contain information about future inflation? *Revista Brasileira de Economia*, SciELO Brasil, v. 67, n. 2, p. 277–286, 2013.

WOODWARD, G. T. The real thing: a dynamic profile of the term structure of real interest rates and inflation expectations in the united kingdom, 1982-89. *Journal of Business*, JSTOR, p. 373–398, 1990.

ANNEX A – Tests for BEI rates

Table 9 – OLS for the BEI rates

	Maturity							
	3	6	9	12	24	36	48	60
c_0	0.53	0.75	0.71	0.38	0.06	0.76	-0.44	0.41
	0.00	0.00	0.00	0.05	0.74	0.00	0.02	0.08
c_1	0.02	0.01	0.02	0.03	0.05	0.02	0.08	0.04
	0.00	0.08	0.10	0.00	0.00	0.06	0.00	0.00
R^2	30.3%	41.0%	29.7%	8.3%	-1.1%	37.8%	12.0%	16.4%
F-test	0.00	0.17	0.26	0.01	0.00	0.00	0.00	0.00

The values below the coefficients are the P-value of coefficients

Table 10 – TSLS for the BEI rates

	Maturity							
	3	6	9	12	24	36	48	60
c_0	0.73	0.83	0.71	0.31	0.09	0.92	-0.61	0.51
	0.00	0.00	0.00	0.14	0.69	0.00	0.00	0.05
c_1	0.01	0.01	0.02	0.04	0.05	0.01	0.09	0.04
	0.04	0.33	0.13	0.00	0.00	0.37	0.00	0.01
R^2	28.5%	39.6%	29.5%	8.1%	-1.3%	37.0%	13.4%	15.0%
F-test	0.04	0.61	0.31	0.01	0.00	0.00	0.00	0.00

The values below the coefficients are the P-value of coefficients

Table 11 – GMM for the BEI rates

	Maturity							
	3	6	9	12	24	36	48	60
c_0	0.56	0.63	0.66	0.34	-0.02	0.84	-0.47	0.42
	0.00	0.00	0.00	0.07	0.90	0.00	0.01	0.06
c_1	0.02	0.02	0.02	0.04	0.06	0.01	0.08	0.04
	0.00	0.01	0.04	0.00	0.00	0.20	0.00	0.00
R^2	30.3%	37.9%	29.7%	8.1%	-3.1%	36.1%	18.2%	15.5%
F-test	0.00	0.04	0.09	0.00	0.00	0.00	0.00	0.00

The values below the coefficients are the P-value of coefficients