Faculdade de Economia,
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Universidade de São Paulo

Texto para Discussão

Série Economia

TD-E 09 / 2013

Natural Resources and GDP

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Valuing Ecosystem Services in Macroeconomic Settings

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1. Introduction

Consider the following questions: In a region where agricultural production is primarily irrigated, how much of its agricultural gross domestic product (GDP) is accounted for by the provisioning services of water? If agriculture in the region is both rain-fed and irrigated, what is the value of rain-water in agricultural production? How much does a coral reef system contribute to a region's GDP? These questions are indicative of those that economists and national account statisticians are beginning to tackle. Answering these questions requires measuring the aggregate flow value of an ecosystem service: water in the first two cases and the tourism services derived from a coral reef system in the last case.

This chapter lays out an approach to numerically calculating the flow (shadow rental) value of an ecosystem service's contribution to aggregate GDP. The approach develops a conceptual framework that is directly linked to an empirical model amenable to numerical solutions. The underlying conceptual model is based on dynamic, general equilibrium theory, and accommodates multiple sectors and multiple regions. In addition to predicting shadow rental values for ecosystem services *over time*, the empirical model can also calculate the unit shadow price (discounted shadow rental values) of the ecosystem service(s) over time.

We focus on the ecosystem services of water primarily because it is a relatively easy service to measure, e.g., the quantity of water used in agricultural and manufactured good production, and because it is a service whose use cuts across most productive sectors in an economy and one with which most readers can identify. Another reason for using water is it is a natural asset that is seldom allocated with markets, and hence, seldom accounted for in GDP. Also, although the ecosystem producing water is complex, if the water comes from a river or rain, and if the water availability is relatively stable, an economist might safely ignore the ecosystem dynamics generating the water and take the quantity of water provided over a period of time as exogenous and given. Hence, for this exercise at least, we can ignore ecosystem dynamics. The choice of Japan as the case study region is driven primarily because the authors are

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familiar with water issues in Japan, and because Japan has readily available national account data and a rich source of land and water use data – requisite data for the empirical model.

As with physical capital assets, an ecosystem service's (or natural asset's) contribution to GDP is given by the value of the flow of services it provides to producers. One major difference between physical (i.e., manmade) capital and a natural asset or ecosystem, is markets typically allocate physical capital, but seldom allocate ecosystem services. In principle, physical capital's contribution to GDP is equal to the unit flow value of physical capital – e.g., the market determined interest rate paid on a unit of capital – multiplied by the size of the capital stock. Here, the interest rate is equal to the revenue an additional unit of capital generates, ceteris paribus. Hence, with physical capital the market captures the flow value of the asset.

The unit flow value of an ecosystem service is usually not valued by a market. Water, for example, is seldom allocated via market mechanisms, and in such cases the flow value of water is not captured by markets. Even though a market might not capture the flow value of water, producers who use water certainly do – earning rent on the water they employ in production. The unit flow value of an additional unit of the ecosystem service, e.g., an additional unit of water, is equal to the additional unit of revenue the producer receives given an additional unit of the ecosystem service. We call this additional revenue the unit *shadow rent* of the ecosystem service, and contrast this with the unit *shadow price* of the ecosystem service – defined here as the discounted present value of current and future unit shadow rents.

In what follows, we link the (regional) shadow rental value of agricultural water to national income by first estimating an agricultural production function and using the production function[3] to estimate water's marginal contribution to agricultural GDP. With this information we disentangle water's contribution to GDP by introducing a factor account for water and readjusting Japan's remaining factor account entries accordingly. Some will view this process as an approach to "greening" national accounts, as it provides analysts with a way to measure part of the ecosystem's direct use value in generating GDP (see World Bank, 2010). Using the re-parameterized agricultural technology we develop an empirical model that predicts a sequence of water and land shadow rental rates for three regions over time. Each sequence of shadow rental values is then used to calculate a sequence of unit shadow prices of the ecosystem service

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[3] See Barbier, 2006, for a discussion on using production functions to value the environment.

over time.

With a unit shadow price in hand, one is then in a position to calculate the value of a stock of natural assets, or possibly the stock value of an ecosystem. The process of measuring the stock value of a natural asset or ecosystem ventures into the literature on national welfare and sustainability, as aggregate measures of welfare and sustainability invariably are based on a measure of the total value of a nation's assets: physical capital, human capital, natural assets (ecosystems) and institutions (see Dasgupta, 2009).

Human wellbeing, i.e., welfare, and its definitions and measurement, is the topic of interest to a wide spectrum of disciplines (see Stiglitz et al., 2008). For the past few decades, GDP and gross national product (GNP) have been popular indices of welfare, where GNP is GDP plus income earned by domestic citizens abroad less income earned by foreign citizens in the region. While GNP is a reasonable measure of economic activity, when measuring welfare in a dynamic setting (e.g., an evolving economy) some economists prefer using wealth or net national product (NNP) as a measure of economic wellbeing or social welfare, where NNP is equal to GNP less depreciation [see Weitzman (1976), Dasgupta and Mäler (2000), Heal and Kriström (2005), and Dasgupta (2009)].

Dasgupta and Mäler (2000) note the NNP concept has been around for over eighty years, but renewed interest in the concept emerged as economists began thinking NNP should be adjusted to reflect the cost of natural resource depletion and degradation, and environmental damages. NNP adjusted to account for such costs is often referred to as "*green-NNP*." They suggest using a wealth based measure of welfare defined as the summed value of natural asset stocks, manmade capital stocks and human capital, where the value of natural asset stocks is the unit (possibly shadow) price of the asset multiplied by the stock of that asset. Heal and Kriström (2005) examine the merits of the Dasgupta and Mäler measure, as well as an income based measure of welfare defined as the present value of consumption at shadow prices, and show (page 1171) that between the wealth and income measures, the income measure "…tracks welfare changes better. Wealth as the value of stocks only tracks welfare if stock prices are constant."

As hinted above, in addition to examining the practical aspects of measuring the value of green account factor entries, this chapter also presents an approach for estimating (actually, calculating) the stock value of a natural asset – a requisite component of aggregate total wealth. The methodology exploits recent advances in dynamic general equilibrium theory and its empirical implantation to calculate the discounted present value of the stream of flow-shadow-values, hence yielding unit values of a natural asset.

We should make one qualification on the unit values we estimate: the shadow value as measured here is only a value of the natural asset or ecosystem as an economically productive factor. If the ecosystem service is associated with externalities – e.g., river flow and habitat preservation valued by society – the approach here will not measure the value of the externality unless the externality is explicitly modeled. Hence, the approach outlined here ***does not try*** to uncover the economic values associated with externalities or natural vistas. Although such exercises are important, we feel they distract from a more fundamental challenge: uncovering a clearer picture of the economic value of natural assets or ecosystem services embedded in GDP – a value typically assigned incorrectly to the factor accounts for labor or capital.

Section 2 discusses input-output data and green accounts, and the role production technologies can play in backing out natural assets' contribution to GDP. Section 3 gives the reader an overview of the economic environment upon which the empirical model and its results are based, and provides a formal definition of the shadow rental value and its corresponding shadow price. Aside from these two definitions and their relationship, all other details of the mathematical model are relegated to the appendix. Section 4 presents the production, consumption and technical change parameter values for Japan, and the empirical model results. Not surprisingly, the empirical simulations reveal shadow rental values can vary widely across regions and over time. More interestingly, the results also suggest factor intensities affect the rate at which shadow rental values evolve over time: the more labor intensive the water using sector is, the slower is the rate of growth in its water shadow rental rate. The results also show the importance of accounting for technical change when deriving shadow prices: ignoring technical change leads to significant underestimates of natural asset shadow prices. Section 4 also presents a measure of the stock value of water in Japan, using 2010 base year prices. Section 5 concludes.

2. Input-Output Data, Green Accounts and Production Technologies – Practical Considerations

Input-output data and production technologies

GDP as measured by value-added is a residual measure, and is the difference between (i) the gross value of all final goods and services produced over a period of time for a given region or economy, and (ii) the value of all intermediate goods used in producing the final goods and services. In the United Nations' System of National Accounts (SNA), final good receipts are summed for a sector and the value of intermediate goods used to produce the final goods are subtracted from it to give sector GDP, sometimes referred to as sector value-added. The SNA then decomposes sector value-added into at least two factor account categories: payments to capital and payments to labor.[4]

In most countries, wage income is relatively straightforward to measure, as are some payments to capital. In Japan, values that cannot be clearly assigned to either a labor or capital account end up in a "mixed income" account – measures of the return to entrepreneurial efforts are included in mixed income, but land rental payments are not. Table 1 presents aggregated "factor accounts" for Tokyo agriculture, manufacturing and services. As suggested above, table 1 has three aggregate factor accounts: labor income, capital income and mixed income. The sum of payments to the labor, capital and mixed income factor accounts is sector value-added. In table 1, the value 15576 is equal to wL_a, where w is the wage rate and L_a is labor demanded: it is the value of wages the agricultural sector paid to labor. The value 6310 is equal to rK_a, where r is the capital rental rate and K_a represents the capital stock level in Tokyo agriculture: it is the value of capital stock rent the agricultural sector paid to owners of capital.

Table 1. Input-Output Table for Tokyo (2008 values)

	Sector		
Factor accts	Agriculture	Manufacture	Service
Capital Income	6310	848946	18106355
Labor Income	15576	5321070	49052716
Mixed (other) Income	20777	874329	19086345
Value-added	42663	7044345	86245416

When constructing an empirical general equilibrium model (static or dynamic), economists often take the data in the input-output (I-O) table and combine it with data on estimates of labor force and capital stock levels to specify sectoral production technologies. The functional forms for these technologies are typically Cobb-Douglas or constant elasticity of substitution (CES) production functions. The Cobb-Douglas function is the simplest functional form to work with because its parameters relate to I-O data in a very straightforward way. For example, the Cobb-Douglas analog to the agricultural sector in table 1 is

$$(1)\quad Y_a = \Psi_a K_a^{\alpha_1} L_a^{\alpha_2} O_a^{\alpha_3}$$

[4] Although the flow of services provided by a natural asset can enter an economy as an intermediate good and/or a primary factor, for the purpose of this chapter we concentrate on its role as a primary factor.

Here Y_a represents agricultural value-added, L_a is the amount of labor employed in agricultural production, while K_a and O_a are indices of the capital stock and other factors, respectively, used in agricultural production.

Under constant returns to scale, the parameters α_1, α_2 and α_3 are a factor's share in the cost of producing the final good is constant, e.g., $\alpha_1 = rK_a/Y_a$. Hence, the parameter α_1 is calculated by dividing the payment to capital, 8533, by agricultural value-added, 42633. Similarly, we get the coefficients for α_2 and α_3 by the respective factor payment by value-added. Then, from table 1, the share of labor costs in producing Tokyo region agriculture is 36.5%, the share of capital is 14.8%, and the share of mixed income is 48.3%. The *scaling parameter*, Ψ_a, is an index of total factor productivity, defined as $\Psi_a = Y_a/\left(K_a^{\alpha_1} L_a^{\alpha_2} O_a^{\alpha_3}\right)$.

Adjusting factor account entries to reflect land and water rent

Having discussed the link between I-O data and production technologies, we now turn attention to the role of natural assets in such a framework. For the discussion at hand, the major problem with equation (1) is it omits the primary factors (natural assets) land and water. A more desirable specification is

(2) $$Y_a = \Psi_a K_a^{\alpha_1} L_a^{\alpha_2} H_a^{\alpha_3} Z_a^{\alpha_4}$$

where H_a and Z_a represents the level of water and land used in agricultural production respectively. Let π represent the unit shadow rent of land and t represent the unit shadow rental value of water. Then, given the structure of agriculture as reflected in equation (2), a more appropriate factor account breakdown is

Table 2. I-O table

	Sector		
Factor accts	Agriculture	Manufacture	Service
Labor Income	wL_a	-	-
Capital Income	rK_a	-	-
Water rent	tH_a	-	-
Land rent	πZ_a	-	-
Value-added	42663		

The principle challenge, now, is to reapportion agricultural value-added over the new factor account mix. One approach to reapportioning GDP over the factor accounts in table 2 is to estimate the parameters of the Cobb-Douglas function as represented in equation (2).[5] In the empirical example that follows, we constructed a panel of data on water use from 1980 through 2006 from Japan Ministry of Agriculture data sources. This data was combined with panel data on cultivated land area, agricultural capital asset values and agricultural labor in man-hours and used to econometrically estimate the factor share parameters in equation (2). Table 3 presents the summary statistics of the data.

Table 3. Summary statistics of regression variables – by region (all values in 1000s)

Region		Real GDP (Yen)	Capital stock	Labor (man hours)	Land (hectares)	Water (m^3)
Chugoku	Mean	498,748,359	4,744,558,394	120,886	3,891	3,136,308
	St dev	50,975,545	1,907,292,245	23,724	820	429,551
Hokkaido	Mean	1,089,496,847	2,259,327,481	150,120	3,349	3,299,005
	St dev	74,413,876	814,582,390	9,227	704	455,029
Hokuriku	Mean	619,986,542	5,815,517,495	64,433	2,524	5,215,874
	St dev	53,159,214	2,590,964,811	8,211	490	465,180
Kanto	Mean	2,191,137,998	37,683,532,501	145,114	13,189	7,715,504
	St dev	183,160,543	17,380,582,383	15,087	2,060	667,786
Kinki	Mean	563,958,471	11,650,957,632	52,263	3,492	2,917,188
	St dev	64,886,850	5,697,336,070	8,538	469	325,167
Kyushu	Mean	1,708,996,737	9,396,824,376	120,788	10,020	5,168,906
	St dev	160,177,522	4,692,235,473	10,775	1,522	603,380
Shikoku	Mean	506,930,618	3,719,117,281	56,374	3,569	1,507,816
	St dev	58,638,174	1,851,387,748	5,179	646	166,844
Tohoku	Mean	1,752,925,545	11,181,298,556	233,162	9,949	10,790,373
	St dev	120,504,985	3,150,428,473	12,850	1,457	1,028,029
Tokai	Mean	991,999,346	14,913,015,784	92,746	5,430	2,763,287
	St dev	102,544,609	6,113,912,699	17,990	813	306,212

To estimate equation (2), we added regional dummy variables to equation (2) to control for regional differences in productivity (and output mix) and added time to control for technical change effects. We also imposed constant returns to scale on the technology by dividing each region's agricultural value-

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[5] A more preferred approach would have been to estimate a production function for each region.

added, capital stock level, man-hour level and water use by its cultivated area, Z_i. The normalized, log-linear version of equation (2) plus the time and dummy variables is presented in equation (4):

(4) $$\ln(y_t) = \Psi + \sum_{i=1}^{9} \beta_i D_i + \alpha_0 time + \alpha_1 \ln(k_t) + \alpha_2 \ln(l_t) + \alpha_3 \ln(h_t) + \varepsilon_t$$

Here Ψ is a constant, α_0 is a technical change parameter, D_i is a regional dummy variable, y_t is output per unit of land, k_t is capital per unit of land, l_t is labor per unit of land, h_t is water per unit of land, and ε_t is the error term.

Equation (4) was estimated using ordinary least squares (OLS). The coefficient estimates for capital, labor, water and time were all significant at the 99% level and had the desired sign. The OLS estimator, however, yielded an R-square of 0.97, suggesting the presence of serial correlation or endogeneity. To correct for potential correlation problems, we estimated both a random-effect and a fixed-effect model, using the 9 region groups as the panel (18 observations per region). The estimated coefficients for both models were very close to those obtained using OLS, while the R-square value dropped to 0.64. Also, the estimated coefficients were very similar regardless of the method used to correct the variance-covariance matrix. Table 4 presents results for the fixed-effect estimation.[6]

Table 4. Regression results for Cobb-Douglas function Fixed-effects (with 9 regional clusters)

ln(y)	Coefficient	Std. error	t-statistic
α_1	0.0519	0.0118	4.39
α_2	0.5848	0.1341	4.36
α_3	0.3094	0.0731	4.23
α_4	0.0539	-	-
time	0.0235	0.0022	10.59
Y	-47.090	4.5770	-10.29
R^2 (overall) = 0.656, F(4,8) = 49.15			

To reapportion the factor payments in table 1 across the table 2 factor categories, simply multiply each factor's share coefficient from table 4 by Tokyo agriculture's value-added. For example, capital's

[6] We also ran the model using non-normalized data – i.e., non-normalized data with land as an additional explanatory variable – and obtained almost identical results.

payment to agriculture is $0.0519 * 42663 = 2214$. Call this new factor payment value the "shadow factor payment" and the corresponding I-O table the "shadow I-O table."

Table 5 presents the original and shadow I-O data for Tokyo agriculture. The shadow results suggest we reapportion all "Other income" and two-thirds of "Capital income" to labor, land and water. After the suggested reapportionment, on average, labor costs account for about 58% of agricultural production value, while (physical) capital rental payments account for a little over 5% of production value. The natural asset, land, accounts for another 5% of production value, while water accounts for about 31% of production value. Hence, about 36% of agricultural value-added accrues to the natural assets (or ecosystem ecosystem services provided by), land and water, with 86% of this being a shadow water rent.

Table 5. Natural asset adjusted I-O accounts for Tokyo agriculture

Factor Accounts	Original I-O Data		"Shadow" I-O Data	
	Level	Share	Level	Share
Capital income	6310	0.148	2214	0.052
Labor income	15576	0.365	24949	0.585
Other income	20777	0.487		
Water rent			13200	0.309
Land rent			2300	0.054
Value-added	42663		42663	

Reconciling the original I-O data with the econometric results raises measurement related questions. For instance, does the "Capital income" account include land rent, and some water rent? If so, then using the econometric results directly could be an acceptable decision. If not, then a better choice would be to leave the capital account alone and reapportion 5 percentage points of "Other income" rent to land, 31 percentage points to water and the remaining 13 percentage points to labor (e.g., as returns to entrepreneurial ability). National account measurement is a challenging task, and the econometric results in table 4 along with the original data in table 2 hint at some of the challenges the SEEA could face when trying to measure natural assets' (or ecosystem services') contribution to sector GDP.

3. Shadow Rental Values and Shadow Prices

Below, we provide a brief description of the modeled economy and relegate the model details to the appendix. Japan is modeled as a small open economy divided into three regions: Tokyo, the rest of Kanto, and the rest of Japan. The country is endowed with four productive factors: capital, labor, land and water, with water and land being regionally specific. Water and land combine to form an ecosystem whose provisioning services contribute to agricultural production. Water is also used in producing manufacturing, residential water and services in Tokyo and the rest of Kanto. The productive factors are used in various combinations across sectors and regions to produce four final goods: agriculture, manufacturing, services in each region, and residential water in Tokyo and the rest of Kanto. Firms produce using constant return to scale technologies, and households make consumption and savings decisions that maximize utility over time. All economic agents interact in a competitive world – i.e. an economy in which there are many buyers and sellers of goods and no one agent can influence prices. Finally, the manufactured and agricultural goods are traded, while the service good and residential water are non-traded.

A complete presentation of the conceptual model is presented in the appendix, and the numerical simulations that follow are based directly upon that model. Before presenting the empirical results, however, we digress to discuss two concepts: shadow rental values and shadow prices.

Shadow rental values

Economic theory suggests producers will use a factor up to the point where the market price (or market rental rate) of the factor is equal to the marginal value product of that factor. If a market does not exist for a factor, e.g., water in Japanese agriculture and manufacturing, the unit (shadow) rental value of water is estimated by calculating the marginal value product of water. Use equation (2) and let p_a represent the unit price of agricultural output. Then the shadow rental value of water, denoted SV_H^a, is defined as

$$SV_H^a(p_a, K_a, L_a, H_a, Z_a) = p_a \frac{\partial Y_a}{\partial H_a} = p_a \alpha_3 \Psi_a K_a^{\alpha_1} L_a^{\alpha_2} H_a^{\alpha_3 - 1} Z_a^{\alpha_4} = \frac{\alpha_3 Y_a}{H_a}$$

while the shadow rental value of land, denoted SV_Z^a, is defined as

$$SV_Z^a(p_a, K_a, L_a, H_a, Z_a) = p_a \frac{\partial Y_a}{\partial Z_a} = p_a \alpha_4 \Psi_a K_a^{\alpha_1} L_a^{\alpha_2} H_a^{\alpha_3} Z_a^{\alpha_4 - 1} = \frac{\alpha_4 Y_a}{Z_a}$$

Equations A.1 – A.3 in the appendix show another way to derive the shadow rental value of water and land using value-added (restricted profit) functions whose argument are own output price, factor prices and the level of natural assets used in production.

Shadow prices

The shadow price of land and water is linked directly to their corresponding shadow rental rates through no-arbitrage conditions. To develop the no-arbitrage condition for land and water, let *Z* represent land and *H* represent water, and define agricultural value added as

$$\Pi(p_a, r, w, H, Z) \equiv \max_{K_a, L_a}\{p_a Y_a - rK_a - wL_a : Y_a = F(K_a, L_a, H, Z)\}$$

Assume an economy only has physical capital, labor, land and water, and denote the economy's endowment of capital and labor by *K* and *L*, respectively. Given the natural assets *H* and *Z*, the total value of physical and natural capital holdings is expressed as

$$A(t) = K(t) + P_z(t)Z + P_H(t)H \tag{5}$$

Accounting for the two natural assets, the flow budget constraint for the economy is given by

$$\dot{K}(t) = r(t)K(t) + w(t)L(t) + \Pi_Z(t) + \Pi_H(t) - E(t) \tag{6}$$

in terms of changes in the capital stock, and

$$\dot{A}(t) = r(t)A(t) + w(t)L(t) - E(t) = rK(t) + rP_z(t)Z + rP_H(t)H + w(t)L(t) - E(t) \tag{7}$$

in terms of asset values. Here, *E*(t) is the value of consumption expenditures, and $\Pi_Z = \frac{\partial}{\partial Z_a}\Pi(\cdot)$ and $\Pi_H = \frac{\partial}{\partial Z_H}\Pi(\cdot)$ are the shadow values of land and water. Next, observe from equations (6) and (7)

$$\begin{aligned} rK + wL - E &= \dot{K} - \Pi_Z - \Pi_H \\ &= \dot{A} - rP_zZ - rP_HH \end{aligned}$$

then take the total derivative of equation (5) with respect to time, and substitute the result into the above equation

$$\dot{K} - \Pi_Z - \Pi_H = \dot{K} + \dot{P}_zZ + \dot{P}_HH - rP_zZ - rP_HH$$

rearrange terms and simplify to get the following arbitrage conditions:

$$r = \frac{\Pi_Z}{P_Z Z} + \frac{\dot{P}_Z}{P_Z} = \frac{\Pi_H}{P_H H} + \frac{\dot{P}_H}{P_H} \tag{8}$$

In the above expression, r represents the return to the household from investing a unit of income in physical capital. The same unit of income can also buy $1/(P_Z Z_a)$ units of land, generating, at time $t + dt$, a rental income of $\Pi_Z/P_Z Z$ plus the rate of change in the land price. If this condition did not hold, optimizing investors could exploit the arbitrage opportunity and move investments out of land and into capital. Given the no-arbitrage conditions hold across natural and physical assets, the time t shadow price of land per unit of labor is given by (see RSS, 2010)

(8) $$p_\zeta(t) = \int_t^\infty e^{-\int_t^\tau [r(\nu)-n-\frac{\dot{p}_k(\nu)}{p_k(\nu)}d\nu} \frac{\Pi_Z(\tau)}{p_k(\tau)} d\tau$$

Here, $p_\zeta = p_Z/p_k$ is the unit shadow price of land relative to the unit price of capital. Equation (8) is a solution to the differential equation defined by (7), and is the discounted present value of all future shadow rents, where the discount factor depends on the rate of return to capital adjusted for depreciation, the rate of growth in the labor force, the rate of exogenous technical change, and the rate of change in the price of (composite) capital. More labor and more efficient labor lends to increased productivity of the natural asset, hence, the impact of n on the shadow price of a natural asset.

4. Empirical model and results

4.1. Model parameters

With the *shadow* factor account data in table 5, one can proceed to construct a dynamic, empirical model to predict shadow water rental rates over time, as well as the unit value of the stock of water over time. The technologies for all but the residential water sector are strictly Cobb-Douglas, while the residential water sector is Cobb-Douglas in capital and labor, but Leontief in water. Preferences are homothetic, with Cobb-Douglas structure. Table 6 presents the factor shares per sector for each region, and the consumption shares for the final goods agriculture, manufacturing and the three service sectors. Preliminary econometric results suggest the elasticity of water in Tokyo and the Rest of Kanto manufacturing is about 5%. We used this information and decreased the capital share of manufacturing for both these regions, yielding the values shown in table 6.

As aggregated, the most capital intensive sectors are the two water supply sectors, followed by the Tokyo and rest of Kanto service sectors. The remaining sectors are quite labor intensive. Growth accounting yields a Harrod neutral rate of technical change equal to $x = 0.018$, and we assume the rate of population and labor force growth equal to $n = 0$. Finally, the felicity function is given by $u(\boldsymbol{q}) = \log \bar{u}$.

Table 6. Factor shares by sector, across regions, and consumption shares for all final goods

			Agriculture	Manufacture	Service			Water Supply	
Factor Shares	Tokyo	Capital Income	0.0519	0.2209	0.4624			0.6050	
		Labor Income	0.5848	0.7391	0.5345			0.3950	
		Water shadow	0.3094	0.0400	0.0031			—	
		Land rent	0.0539	—	—			—	
	Rest of Kanto	Capital Income	0.0519	0.3453		0.4189			0.624
		Labor Income	0.5848	0.6147		0.5762			0.375
		Water shadow	0.3094	0.0400		0.0048			—
		Land rent	0.0539	—		—			—
	Rest of Japan	Capital Income	0.0519	0.3692			0.3597		—
		Labor Income	0.5848	0.6308			0.6403		—
		Water shadow	0.3094	—			—		—
		Land rent	0.0539	—			—		—
Consumption shares			0.0120	0.1634	0.0206	0.023	0.1199	0.0007	0.0006

4.2. Empirical results: Water shadow rental values and shadow prices

Table 7 lists the unit water shadow rental values for all sectors except residential water. The values show water rents can vary significantly across regions and sectors, and also can exhibit non-monotonic behavior. Water rent for Tokyo manufacturing falls for about 20 years, and then begins increasing again. This is because Tokyo manufacturing is the most labor intensive of all sectors, and increasing wages combined with falling capital rental rates makes it difficult for the sector to compete for resources. Increased labor productivity eventually dominates the wage and capital rental rate effects, leading to increased water rents.

Table 7. Unit shadow water rental values

	Agriculture			Manufacturing		Services	
Year	Tokyo	ROK	ROJ	Tokyo	ROK	Tokyo	ROK
2008	93.88	41.02	69.52	311.14	52.44	188.76	80.06
2018	97.76	42.71	72.39	147.87	58.94	274.27	116.57
2028	111.04	48.52	82.23	125.00	68.90	353.48	150.36
2038	130.12	56.85	96.36	129.81	81.69	436.21	185.61

2048	154.37	67.45	114.32	146.35	97.40	528.98	225.12
2058	184.10	80.44	136.34	170.78	116.41	636.78	271.01
2068	220.05	96.14	162.96	202.22	139.26	764.17	325.23
2078	263.25	115.02	194.96	240.95	166.67	915.80	389.77
2088	315.07	137.67	233.33	287.88	199.51	1096.90	466.85
2098	377.16	164.79	279.31	344.35	238.85	1313.47	559.03
2108	451.52	197.28	334.38	412.10	285.95	1572.64	669.33

Figure 1 reveals the more capital intensive is a sector, the larger is the rate of growth in its water shadow value over time.[7] Results in Roe, Smith and Saraçoglu (2009) suggest, in the process of economic growth, capital deepening occurs and tends to favor the more capital intensive sectors of an economy as it evolves. In other words, capital deepening makes capital intensive sectors more competitive in factor markets. They refer to this competitiveness effect as a *Rybczynski-type effect*, suggesting that capital deepening tends to have an effect similar to that expressed in the Rybczynski theorem.[8] With capital deepening, the rate of return on (or unit cost of) capital falls over time, while the wage rate increases over time. This Rybczynski-type effect results in the more capital intensive sectors garnering a larger share of resources over time, and hence, producing a larger share of GDP over time.

[7] The value for the Tokyo and ROK service sectors are not identical, but close enough to warrant plotting only one of price series

[8] The Rybczynski theorem uses a two-output, two-input model to show when the level of a factor increases, the output of the sector using that factor most intensively increases, while the output of the competing sector decreases.

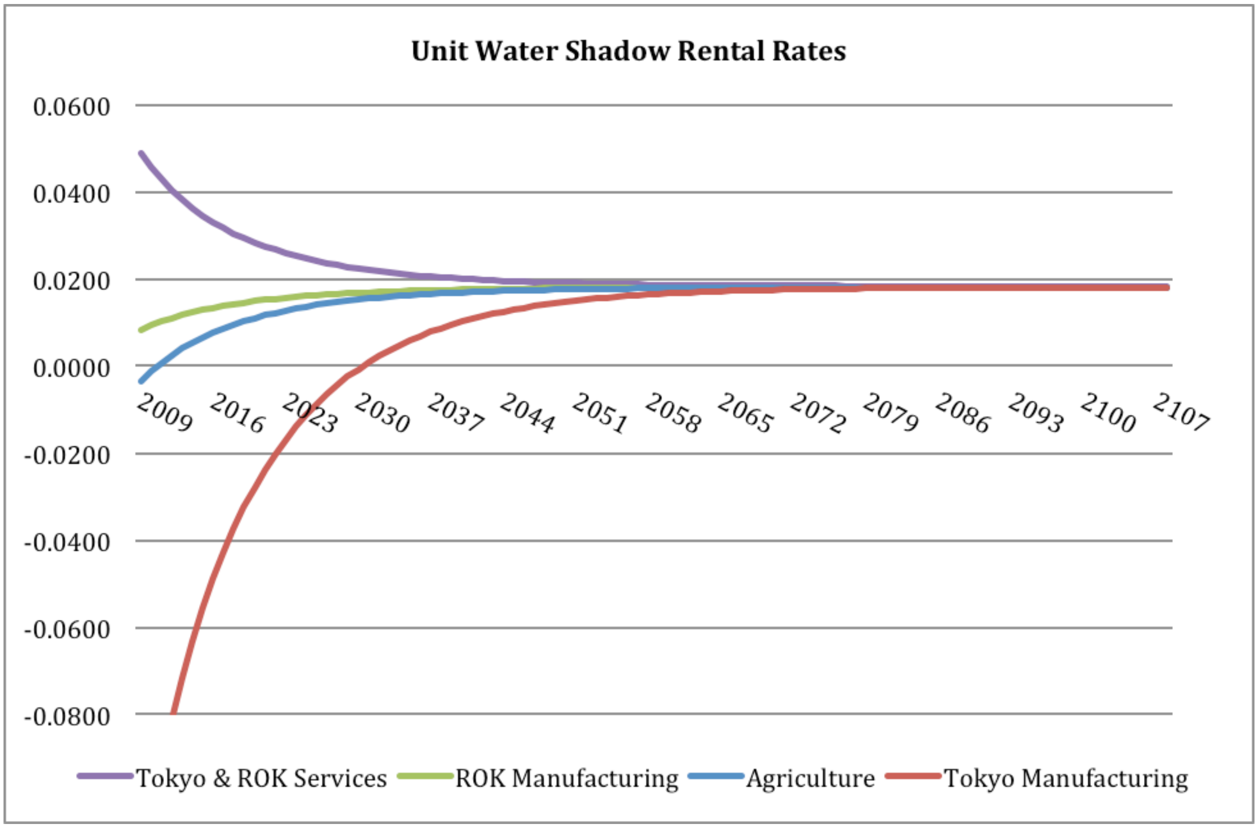

Figure 1. Sector shadow rental values

The initial question raised in the introduction is how much, or what share, of GDP is attributable to water? With linearly homothetic technologies and sector factor shares, the answer is readily addressed. The Cobb-Douglas technologies tell us the share of GDP coming from land and water in agricultural production is equal to land and water's agricultural cost share values multiplied by agriculture's share of GDP. Table 8 presents the trajectory of natural asset shares of GDP over a 60 year period. Natural assets' share of GDP in Japan is very small, and falls slightly over time. These values would increase if we disentangled water's contribution to the rest of Japan's manufacturing and service sectors. The increase, however, is not likely to surpass 1.5 to 2 percent of GDP. Still, the value of this small share is larger than the GDP of over 120 developing countries.

Table 8. The share of natural asset value embedded in GDP

Year	2008	2018	2028	2038	2048	2058	2068	2078
GDP Share	0.41%	0.33%	0.31%	0.31%	0.30%	0.30%	0.30%	0.30%

With homothetic technologies, land shadow values will behave analogously to that of water. As such, we do not discuss the land rent dynamics nor present their numerical results.

4.2.1. Ecosystem service shadow prices

Combining equation (8) with the transition interest rate values and transition unit water shadow rental rates allows for the calculation of unit water shadow prices. Table 9 lists the unit water shadow prices across sectors and over time.

Table 9. Unit shadow water price values

	Agriculture			Manufacturing		Services	
Year	Tokyo	ROK	ROJ	Tokyo	ROK	Tokyo	ROK
2008	2095.19	915.46	1551.62	3017.92	1280.23	6298.74	2678.54
2018	2783.68	1216.28	2061.49	2949.86	1739.55	9156.58	3895.78
2028	3466.73	1514.73	2567.33	3352.28	2183.59	11792.33	5018.21
2038	4218.10	1843.03	3123.77	3942.29	2665.25	14545.76	6190.45
2048	5084.60	2221.63	3765.47	4686.96	3216.98	17635.18	7505.53
2058	6105.16	2667.54	4521.26	5595.13	3864.84	21227.06	9034.37
2068	7318.33	3197.62	5419.69	6690.36	4633.95	25472.08	10841.16
2078	8766.27	3830.27	6491.97	8005.53	5551.35	30525.67	12992.05
2088	10497.39	4586.65	7773.98	9582.03	6647.91	36560.95	15560.75
2098	12568.63	5491.64	9307.87	11470.40	7959.76	43778.49	18632.62
2108	15047.56	6574.77	11143.67	13731.55	9529.75	52414.89	22308.38

Except for a 10 year period for Tokyo manufacturing, the water shadow prices for each sector increases over time. Although not reported here, converting each of the prices to present values, however, yield a price trajectory for Tokyo manufacturing that falls over time. Also, not reported are the shadow price growth rates across sectors: as with the water shadow rental values, the more capital intensive is a sector, the higher is the rate of growth in its unit water shadow price.

At this point we investigate the potential impact of deviating from theory when deriving empirical results. We consider two cases. First, we calculate shadow prices using an approach used in most empirical exercises:

$$Price_{asset}(t) = \frac{Rent_{asset}(t)}{r(t)}$$

Here, $Price_{asset}(t)$ is the time-t shadow price of an asset, $Rent_{asset}(t)$ is the time-t rental rate, and $r(t) = r^k(t) - \delta$ is the rate of return on capital adjusted for depreciation. Second, we calculate shadow prices while ignoring technical change. Increases in the effective labor force – either by an increase in the number of units of labor or an increase in the productivity of labor via technical change – leads to an increase in the productivity of the asset, and hence its shadow rental rate and its (shadow) price. Also, an increase in the unit value of (composite) capital puts upward pressure on the natural asset (shadow) price, via arbitrage forces. Table 10 presents the water shadow price for manufacturing and services calculated using the three different calculations.

Table 10. Unit shadow water price values

Year		Manufacturing				Services			
		Tokyo	% Difference	ROK	% Difference	Tokyo	% Difference	ROK	% Difference
2008	Mod1	3018		1280		6299		2679	
	Mod 2	1570	-48%	1035	-19%	5664	-10%	2399	-10%
	Mod 3	4057	34%	684	-47%	2461	-61%	1044	-61%
2038	Mod1	3942		2665		14546		6190	
	Mod 2	700	-82%	1397	-48%	9421	-35%	3995	-35%
	Mod 3	1534	-61%	965	-64%	5154	-65%	2193	-65%
2068	Mod1	6690		4634		25472		10841	
	Mod 2	667	-90%	1441	-69%	9941	-61%	4216	-61%
	Mod 3	1439	-78%	991	-79%	5439	-79%	2315	-79%

The results along the row "Mod 1" are values calculated using the "correct" pricing model, as represented by equation (8). The results along the row "Mod 2" are values calculated when ignoring technical change. The results along the row "Mod 3" are values calculated using the "standard" approximation used [FIND CITATIONS] in the literature. Both Mod 2 and Mod 3 yield results that deviate from the results one obtains when implementing the theoretical model correctly. In Mod 2, ignoring the impact of improving labor productivity introduces a serious bias in the shadow price predictions. This effect will likely hold for most empirical exercises. The empirical model here, as with the Solow model, predicts the rate of return to capital will fall as capital deepening occurs. The standard model underestimates shadow prices, as it does not accommodate a fall in interest rates over time. Dividing the current rental rate by the rate of return on capital 50 or 60 years later yields unit water shadow prices closer to those estimated using the correct pricing model. Hence, ignoring technical change and using the standard approximation leads to, what could be, seriously biased estimates of natural asset and ecosystem unit shadow prices.

4.2.2. Natural assets and wealth

The last question of interest is: what is the value of the stock of water and land, and how large is this value relative to the value of the stock of physical capital? Since water is ignored in the rest of Japan, we focus attention on the natural and physical asset values in the Kanto region. The price of capital in the initial period, 2008, is numeraire, and the stock of physical capital in Kanto is estimated at K_{2008} = 4,620,080 billion. The value of Kanto agricultural land in 2008 is 1272.98 billion Yen. Finally, the average unit shadow price of Kanto water in 2008 is equal to 4378 Yen per cubic meter, while a lower bound on the estimate of Kanto water stocks is 251.9 billion cubic meters[9]. Then the total value of the ecosystem services produced by land and water in the Kanto Plain is equal to 1,103,256 billion Yen, suggesting the stock value of Kanto land and water is about one forth that of its physical capital. This ratio is similar to the ratio of natural to man-made capital for the world as measured by the World Bank (see Table 2.3 in World Bank, 2006).

5. Conclusion

This chapter lays out a methodology for measuring a natural asset's or ecosystem service's contribution to GDP – their shadow rental values – and then projecting those values over time. The approach uses production theory, and its empirical implementation hinges on the ability to identify the parameters of a production technology for the sector using the ecosystem service (see Barbier, 2007, 2009). Using appropriately estimated production technologies allows for spatial and sector differences in water productivity across an economy, which in turn can be used in a dynamic general equilibrium modeling framework having equilibrium factor prices and other endogenous variables that evolve over time.

These sequences of shadow rental values are then used to calculate the unit shadow stock value of the ecosystem service. The approach is perhaps more general than a reader might guess, as it can accommodate non-competitive factor or output market structures, and admits a variety of functional forms for consumers and producers. The approach can also introduce ecosystems dynamics: e.g., ecosystem services deriving from a large groundwater aquifer plus surface water sources like that observed in the Punjab, India.

The discussion in this chapter focused on backing out the value of natural assets or ecosystem services embedded in GDP. Using multi-output technologies, one could follow a similar approach and measure the economy-wide or regional value of externalities, e.g., the shadow flow value of carbon storage in the event markets for such services do not exist. These values will not enter GDP, but the discounted flow values can be used to calculate the stock value of carbon, and hence, augment the value of natural assets in wealth accounting exercises.

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[9] Here, water stock levels are measured as the quantity of water used in Kanto agriculture, manufacturing, services and residential water. The actual stock of Kanto water is stored in the relatively vast system of dams and aqueducts – values

This chapter describes one direction economists can take in measuring economy-wide flow and stock values of natural assets and ecosystems. Certainly other approaches to measuring these values exist. The integrated global models, MIMES and GUMBO[10] are examples of models that integrate ecosystems and economics and generate values for ecosystem services. Noted shortcomings of both these models are: (i) neither MIMES nor GUMBO include market allocation mechanisms for allocating capital and labor across competing sectors within a region; and (ii) the rate at which capital accumulates – and indirectly, the rate at which the economy grows – is exogenous: the rate at which capital accumulations within a sector is chosen by the modeler (Boumans et al., 2002). Hence, the models have undesirable allocation features, and although touted as dynamic models, the economic side of these models is not dynamic. This appears to be the case with most "dynamic" models that integrate ecosystems or natural resources, and economics (see, for example, Chen et. al (2002). One exception is the DICE model (Nordhaus, 1993). While dynamic, the DICE model does not accommodate natural capital, and hence, is not designed to yield values of natural capital or ecosystem services.

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[10] See Boumans et al, 2002, for a discussion of the multiscale integrated earth systems model (MIMES) and the Global Unified Metamodel of the Biosphere (GUMBO).

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Appendix

The economic environment: The economic model takes as its point of departure, the three sector, small open economy model in chapter 4 of Roe, Smith, and Saraçoglu (2009). We represent Japan as a small, open economy and divide it into three regions: Tokyo, the rest of Kanto, and the rest of Japan – indexed by i = 1, 2 and 3 respectively. Each region has an agricultural, manufacturing and service sector, indexed by j = a, m and s respectively. In addition, Tokyo and the rest of Kanto have a municipal water authority that provides water to their households and service sector, with sector index $j = h$. The agricultural output from each region is a perfect substitute in consumption, as is the manufacturing output of each region. Service sector outputs are not substitutes for one another in consumption: municipal water also does not serve as substitutes.

Denote the time t production of residential water, the agricultural, manufacturing and service goods respectively by $Y_{\omega i}(t), Y_{ai}(t)$, $Y_{mi}(t)$ and $Y_{si}(t)$. In what follows, the time t notation is suppressed: e.g., we typically represent the time t production of Kanto-1 agriculture by Y_{a1}, instead of $Y_{a1}(t)$. Agricultural output is a consumption good consumed within the country or traded internationally at given world prices. For each region, residential water is a pure consumption good only consumed within the region with endogenous (regional) final good prices. The service sector output of each region is either consumed or saved, but not traded. Hence, the service good price and the water shadow value(s) are endogenous to each region. Manufacturing output from each region is either consumed or saved, with an exogenously determined (world) final good price. Service sector and manufacturing output are combined to produce a composite capital: an index of aggregate capital. In what follows, composite capital, denoted $Y_k(t)$, is a least cost combination of the three regional manufactured goods, Y_{m1}, Y_{m2} and Y_{m3}, and the three regional service sector goods, Y_{s1}, Y_{s2} and Y_{s3}. The accumulation of composite capital, net of depreciation, yields capital stock, the services of which are employed in producing the agricultural, manufacturing, and service goods. The value of water, composite capital, and land are assets held by households.

Let H_{ji} the water endowment of sector-j in region-i, and let Z_i denote region-i's agricultural land endowment. Assume both the stock of land and available water remains constant over time. Each region's land endowment is a factor specific to agriculture in that region. H_{ji} is also a region/sector specific factor, while labor and composite capital are mobile across regions and sectors. Subsequent models allow water to move across sectors within a region or across regions. Let L denote the stock of labor, also assumed constant over time. For each region, the services of labor, composite capital, and the region's land and water endowment are employed to produce the agricultural, manufacturing, residential water and service goods.

At each instant in time, household income derives from: (i) providing labor services L in exchange for wages $w(t)$, (ii) earning interest income at rate $r^k(t)$ on capital assets $K(t)$, (iii) receiving rent $\tau_i(t)$ from land Z_i, and (iv) capturing rent from the i^{th} region's water resources H_{ji}, with unit water rent denoted $v_{ji}(t)$. The representative agent uses income to invest in composite capital and purchase final consumption goods. Denote the level of total household consumption of the agricultural and manufactured good by the scalar values $Q_a(t)$ and $Q_m(t)$ respectively, the level of service sector output by $Q_s(t)$, and the level of residential water consumption (in region i) by $Q_{hi}(t)$. The initial capital stock, denoted K_0, is given, and the initial endowment of labor, L_0, is normalized to unity, as are the regional land endowments and traded good prices: i.e., $Z_i = p_a = p_m = 1$. We assume labor force growth is negligible, and hence aggregate labor supply is given by $L(t) = L = L_0$. The non-traded good price for each region i is endogenous and denoted p_{si}, $i = 1,2,3$.

Production: Let $K_{ji}(t)$ denote the time t level of capital stock employed in producing good-ji, and firms in each sector employ a constant returns to scale (CRS) technology. The aggregate technology for services in the rest of Japan is represented by

$$Y_{s3}(t) = F^{j3}\left(K_{j3}, A(t)L_{j3}\right)$$

The function $A(t)$ represents the exogenous level of growth in labor productivity. The production functions $F^{j3}(\cdot)$ – and all production technologies below – are twice continuously differentiable, non-decreasing and strictly concave in all respective arguments, and satisfy the standard Inada conditions. The cost function corresponding to $F^{s3}(\cdot)$ is

$$C^{s3}\left(r^k, w\right)Y_{s3} \equiv \min_{K_{s3},L_{s3}} \{r^k K_{s3} + wL_{s3}: Y_{si} = F^{si}\left(K_{ji}, AL_{ji}\right)\}$$

The technology for municipal water provision in Kanto-1 and Kanto-2 is represented by

$$H_{hi} \leq \min_{L_{hi}, K_{hi}} \left\{ F^{hi}\left(K_{hi}, A(t) L_{hi}\right), \frac{H_{hi}}{\sigma_{hi}} \right\}, \; i = 1,2$$

where the parameter σ_{hi} is the input-output coefficient that determines the amount of "river" water required to produce one unit of residential water: here, we assume $\sigma_{hi} = 1$. The cost functions for municipal water provision are given by

$$C^{hi}(r^k, w) H_{hi} \equiv \min_{K_{hi}, L_{hi}} \{ r^k K_{hi} + w L_{hi} : H_{ai} = F^{hi}(K_{hi}, A L_{hi}) \}$$

Given the curvature properties of the technologies, $F^{ji}(\cdot)$, each unit cost function is concave and linearly homogeneous in input prices, and satisfy Shepard's lemma.

The aggregate technologies for services in Kanto-1 and Kanto-2, and manufacturing in each region are represented by

$Y_{ji}(t) = F^{ji}\left(K_{ji}, A(t) L_{ji}, \Lambda(t) H_{ji}\right)$: $j = m$ and $i = 1,2,3$; $j = s$ and $i = 1,2$

Here, the function $\Lambda(t)$ represents the exogenous level of growth in water productivity. Thus, in addition to labor augmenting technological change, technological change in the employment of water in each region also occurs. The dual analog of these manufacturing and service technologies is the value-added function defined as

(A.1) $\Pi^{mi}\left(p_m, r^k, w, H_{mi}\right) \equiv \max_{K_{mi}, L_{mi}} \left\{ p_m Y_{mi} - r^k K_{mi} - w L_{mi} : Y_{mi} = F^{mi}(K_{mi}, A L_{mi}, \Lambda H_{mi}), i = 1,2,3 \right\}$

and

(A.2) $\Pi^{si}\left(p_{si}, r^k, w, H_{si}\right) \equiv \max_{K_{si}, L_{si}} \left\{ p_{si} Y_{si} - r^k K_{si} - w L_{si} : Y_{si} = F^{si}(K_{si}, A L_{si}, \Lambda H_{si}), i = 1,2 \right\}$

The aggregate technologies for agricultural production in each region is represented by

$$Y_{ai}(t) = F^{ai}(K_{ai}, A(t) L_{ai}, \Lambda(t) H_{ai}, B(t) Z_i), \qquad i = 1,2,3$$

where the function $B(t)$ represents the exogenous level of growth in land productivity. For each region-i, the agricultural value-added function is defined as

(A.3) $$\Pi^{ai}(p_a, r^k, w, H_{ai}, Z_i) \equiv \max_{K_{ai}, L_{ai}} \{p_a Y_{ai} - r^k K_{ai} - w L_{ai} : Y_{ai} = F^{ai}(K_{ai}, AL_{ai}, \Lambda H_{ai}, BZ_i)\}$$

Finally, given the properties of $F^{ji}(\cdot)$, each value-added function is concave in the wage rate, the rate of return to capital, and own price, and satisfies Hotelling's Lemma. Furthermore, constant returns to scale in the inputs yields value-added functions that are separable in prices and the fixed factors. For example, the value added function $\Pi^{h1}(p_{h1}, r^k, w, H_{h1})$ is concave in prices, and can be written as $\pi^{h1}(p_{h1}, r^k, w)H_{h1}$ where $\pi^{h1}(p_{h1}, r^k, w)$ is the unit shadow value of Kanto-1 municipal water.

We ignore the water endowment for manufacturing and services in the rest of Japan and set the respective values equal to zero. We also implicitly aggregate the municipal water sectors in the rest of Japan with ROJ services.

Composite capital: In Japan, over 98% of savings comes from the manufacturing and service sectors. To accommodate this feature of the economy, we assume the capital stock is created by combining the saved output of the saving sectors, and call the result composite capital, denoted $Y_k(t)$. Composite capital production is governed by the CRS Cobb-Douglas technology:

$$Y_k = \Upsilon_{m1}^{\zeta_{m1}} \Upsilon_{m2}^{\zeta_{m2}} \Upsilon_{m3}^{\zeta_{m3}} \Upsilon_{s1}^{\zeta_{s1}} \Upsilon_{s2}^{\zeta_{s2}} \Upsilon_{s3}^{\zeta_{s3}}, \ 0 < \zeta_{ji} < 1$$

where $\Upsilon_{ji}(t)$ is the time *t* level of manufacturing or service sector output used in producing composite capital. The composite capital good's corresponding cost function is given by:

$$c^k(p_{m1}, p_{m2}, p_{m3}, p_{s1}, p_{s2}, p_{s3})Y_k = \prod_{j=m,s} \prod_{i=1}^{3} \zeta_{ji}^{-\zeta_{ji}} p_{ji}^{\zeta_{ji}} Y_k = p_k Y_k,,$$

where $c^k(\cdot)$ is with the unit cost of composite capital and is equal to p_k in equilibrium. Since the service sector prices p_{si} are endogenous and evolve over time, it follows that in equilibrium, p_k evolves over time also. The composite capital factor demand function for manufacturing output Y_m is obtained using Shepard's lemma. Note, that if Y_k units of composite capital are produced, the aggregate stock of capital increases by Y_k. Hence, in equilibrium, the instantaneous change in the aggregate stock of capital, denoted K, is given by $\dot{K} = Y_k$ adjusted for loses due to depreciation.

The savings and consumption behavior of households: Let $q_{ji}(t) = Q_{ji}(t)/L$, and define the time t per capita consumption vector as
$q(t) = (q_a, q_m, q_{h1}, q_{h2}, q_{s1}, q_{s2}, q_{s3})$

The present value of intertemporal utility is a time-separable weighted sum of all future utility flows

(5) $$U = \int_0^\infty u\big(q(t)\big)\, e^{-\rho t}$$

where $\rho > 0$ is the discount rate of future consumption. We assume the felicity function $u(\cdot)$ is homothetic, twice continuously differentiable, increasing and strictly concave in each argument.

Given prices $p(t) = (p_a, p_m, p_{h1}, p_{h2}, p_{s1}, p_{s2}, p_{s3})$, the minimum expenditure capable of yielding welfare level $\bar{u}(t)$ per household member is given by

(6) $$\epsilon(p, \bar{u}) = \chi(p)\bar{u} \equiv \min_q\{p \cdot q : \bar{u} \le u(q)\}$$

The properties of $u(\cdot)$ imply the expenditure function is increasing and concave in p, increasing in u, and satisfies Shepard's lemma.

A flow budget constraint expresses time t savings as the difference between income and expenditures. Let τ_i denote rent per effective unit of land BZ_i in region i, and let v_{ji} denote the rents (shadow value) per effective unit of water ΛH_{ji} in sector j of region i. Income is derived from labor income, wL, returns to the capital asset, $r^k K$, returns to land assets in each region, $B\sum_{i=1}^{3}\tau_i Z_i$, and returns to water as measured by the shadow value of water in the two regions, $\Lambda\sum_{i=1}^{2}\sum_{j=a,h,m,s} v_{ji}H_{ji}$. Thus, as modeled, the rents to the government's allocation of water accrue to households. Then the representative household's flow budget constraint in per worker terms is expressed as

(7) $$\dot{k}(t) = \frac{1}{p_k}\left[w + r^k k + \sum_{i=1}^{3}\big(\tau_i \tilde{B} Z_i + v_{ai}\tilde{\Lambda} H_{ai}\big) + \Lambda \sum_{i=1}^{2}\sum_{j=a,h,m,s} v_{ji}H_{ji} - \epsilon\right] - \delta k$$

Here $\dot{k}(t) = K(t)/L$ is household saving, while $\tilde{B}(t) = B(t)/L$ and $\tilde{\Lambda}(t) = \Lambda(t)/L$. The representative household chooses the sequence of consumption bundles $\{\boldsymbol{q(t)}\}_{t\in[0,\infty)}$ to maximize intertemporal utility (5) subject to the flow budget constraint (7). A solution to the present value Hamiltonian derived from equations (5) and (7) is the Euler equation

(8) $$\frac{\dot{\epsilon}}{\epsilon} = \frac{r^k}{p_k} - \delta - \rho + \frac{\dot{p}^k}{p_k}$$

Given, $L = 1$, the initial capital stock is given by

(9) $$k(0) = K_0$$

and the transversality condition satisfies

(10) $$\lim_{t \to \infty} \lambda(t)k(t) = 0$$

Here, $\lambda(t)$ is the costate variable for the equation of motion, and is the present value shadow price of income. Together, equations (7) – (10) characterize the representative household's optimization problem.

The empirical model includes exogenous technical change. Harrod neutral, labor augmenting technical change is introduced into the model by redefining labor in terms of effective labor, $\eta(\mathrm{t})L(\mathrm{t})$, where $\eta(t) = e^{xt}$, and x is the Harrod neutral rate of technical change. Accompanying the introduction of exogenous technical change, are changes in the Euler equation and flow budget constraint: both expressions need to be specified in units of effective labor. The first step in doing this is to specify expenditure $\epsilon(\cdot)$ in per-effective-labor units: i.e., $\hat{\epsilon}(\cdot) = \epsilon(\cdot)e^{xt}$. Then,

$$\frac{\dot{\hat{\epsilon}}}{\hat{\epsilon}} = \frac{\dot{\epsilon}}{\epsilon} - x$$

and the Euler equation (8) becomes

(11) $$\frac{\dot{\hat{\epsilon}}}{\hat{\epsilon}} = \frac{r^k}{p_k} - \delta - \rho - x + \frac{\dot{p}^k}{p_k}$$

and the flow budget constraint becomes

(12) $$\dot{\hat{k}}(t) = \frac{1}{p_k}\left[\hat{w} + r^k\hat{k} + \sum_{i=1}^{3}\left(\tau_i\hat{B}Z_i + v_{ai}\hat{\Lambda}H_{ai}\right) + \Lambda\sum_{i=1}^{2}\sum_{j=a,h,m,s} v_{ji}\hat{\Lambda}H_{ji} - \epsilon\right] - \delta\hat{k}(\delta + x)$$

where $\hat{w} = we^{xt}, \hat{k} = ke^{xt}, \hat{B} = \tilde{B}(t)e^{xt}$ and $\hat{\Lambda} = \tilde{\Lambda}(t)e^{xt}$.

Equilibrium: For all intents and purposes, equilibrium is a prediction of how the economy will perform, given the economic environment, and its primitives and factor endowments. We define equilibrium as follows: Given water assignments

$$\{H_{a1}, H_{a2}, H_{a3}, H_{m1}, H_{m2}, H_{s1}, H_{s2}, H_{h1}, H_{h2}\},$$

a competitive equilibrium is an endogenous sequence of capital stock and expenditure levels $\{\hat{k}(t), \hat{\epsilon}(t)\}_{t\in[0,\infty)}$ and a nine-tuple sequence of positive values $\{\hat{w}(t), r^k(t), p_{s1}(t), p_{s2}(t), p_{s3}(t), p_{h1}(t), p_{h2}(t), \hat{y}_{m3}(t), \hat{y}_{s3}(t)\}_{t\in[0,\infty)}$

such that the representative household's utility is maximized and at each *t*, the following intra-temporal conditions are satisfied:

1. Zero profit in ROJ manufacturing and services

(13) $$C^{m3}(r^k, \hat{w}) = p_m$$

$$C^{s3}(r^k, \hat{w}) = p_{s3}$$

and zero profit in Tokyo and ROJ municipal water provision

(14) $$C^{hi}(r^k, \hat{w}) = p_{hi}, \quad i = 1,2$$

2. Labor market clearing

(15) $$-\sum_{i=1}^{3} \frac{\partial \Pi^{ai}(p_a, r^k, \hat{w}, H_{ai}, Z_i)}{\partial \hat{w}} - \sum_{i=1}^{2} \frac{\partial \Pi^{si}(p_{si}, r^k, \hat{w}, H_{si})}{\partial \hat{w}} - \sum_{i=1}^{2} \frac{\partial \Pi^{ai}(p_m, r^k, \hat{w}, H_{mi})}{\partial \hat{w}} + \sum_{i=1}^{2} \frac{\partial C^{hi}(r^k, \hat{w}) H_{hi}}{\partial \hat{w}} + \sum_{j=m,s} \frac{\partial C^{j3}(r^k, \hat{w}) \hat{y}_{j3}}{\partial \hat{w}} = 1$$

2. Capital market clearing

(15)

$$-\sum_{i=1}^{3} \frac{\partial \Pi^{ai}(p_a, r^k, \hat{w}, H_{ai}, Z_i)}{\partial r^k} - \sum_{i=1}^{2} \frac{\partial \Pi^{si}(p_{si}, r^k, \hat{w}, H_{si})}{\partial r^k} - \sum_{i=1}^{2} \frac{\partial \Pi^{ai}(p_m, r^k, \hat{w}, H_{mi})}{\partial r^k} + \sum_{i=1}^{2} \frac{\partial C^{hi}(r^k, \hat{w}) H_{hi}}{\partial r^k} + \sum_{j=m,s} \frac{\partial C^{j3}(r^k, \hat{w}) \hat{y}_{j3}}{\partial r^k} = \hat{k}$$

where $\hat{y}_{ji} = \frac{Y_{ji}}{Le^{xt}}$, $j = m, s$, and $i = 1,2,3$.

The non-traded good market clears in each region

(16) $$\frac{\partial \epsilon(\cdot)}{\partial p_{si}} + \hat{y}_{ski} = \frac{\partial \Pi^{si}(p_{si}, r^k, \hat{w}, H_{si})}{\partial p_{si}}, \quad i = 1,2$$

(17) $$\frac{\partial \epsilon(\cdot)}{\partial p_{s3}} + \hat{y}_{sk3} = \hat{y}_{s3}$$

where $\hat{y}_{ski} = \frac{Y_{si}}{Le^{xt}}, i = 1,2,3$.

If a solution to the system (13) – (17) exists, it will be an eleven-tuple sequence of endogenous variables, with each variable being a function of the exogenous variables $\{p_a, p_m, Z_1, Z_2, Z_3, H_{a1}, H_{a2}, H_{a3}, H_{m1}, H_{m2}, H_{s1}, H_{s2}, H_{h1}, H_{h2}\}$

and the remaining endogenous variables $\{\hat{k}, \hat{\epsilon}\}$. In practice, however, solving the system is facilitated by representing the endogenous variables $\{\widehat{w}(t), r^k(t), p_{h1}(t), p_{h2}(t), \hat{y}_{m3}(t), \hat{y}_{s3}(t), \hat{\epsilon}\}$ as a function of p_{s1}, p_{s2}, p_{s3} and $\hat{k}$. Hence, the solution can be identified with four differential equations. To conserve on space we leave these derivations to the motivated reader.