Preferências do Banco Central do Brasil sob o regime de metas de inflação: estimação usando um modelo DSGE para uma pequena economia aberta.

Profa. Dra. Andreza Aparecida Palma
A série TEXTO PARA DISCUSSÃO tem como objetivo divulgar: i) resultados de trabalhos em desenvolvimento na FEA-RP/USP; ii) trabalhos de pesquisadores de outras instituições considerados de relevância dadas as linhas de pesquisa da instituição. Veja o site da Comissão de Pesquisa em www.cpq.fearp.usp.br. Informações: e-mail: cpq@fearp.usp.br
RESUMO: O objetivo principal deste trabalho é estimar as preferências do Banco Central do Brasil no período pós metas de inflação (janeiro 2000 a junho de 2011), usando um modelo DSGE com fundamentos microeconômicos para uma pequena economia aberta, tomando como base especialmente o trabalho de Kam, Lees e Liu (2009). Na literatura brasileira, há dois trabalhos principais sobre o tema: Aragon e Portugal (2009), que utiliza um exercício de calibração e estimação por máxima verossimilhança em um modelo backward-looking para a estimação das preferências, e Palma e Portugal (2011), que usa um modelo novo-keynesiano padrão com expectativas racionais para uma economia fechada e estimação através do filtro de Kalman e o método da máxima verossimilhança. O modelo utilizado neste trabalho considera que o Banco Central minimiza uma função perda, levando em consideração o desvio da inflação em relação a meta, a estabilização do produto, a suavização da taxa de juros e, distintamente dos trabalhos anteriores, a taxa de câmbio. Os resultados permitem afirmar que a maior preocupação da autoridade monetária no período foi com a estabilização da inflação, seguida pela suavização da taxa de juros, estabilização do produto e, por último, a estabilização da taxa de câmbio.

Palavras-chave: política monetária, preferências do Banco Central do Brasil, estimação bayesiana, DSGE.

JEL: C11, E12, E52, E61

RESUMEN: El objetivo principal de este trabajo es estimar las preferencias del Banco Central de Brasil en el periodo pos metas de inflacion (enero 2000 a junio de 2011), usando un modelo DSGE con fundamentos microeconomicos para una economia abierta pequena, tomando como base especialmente el trabajo de Kam, Lees y Liu (2009). En la literatura brasileña, hay dos trabajos principales sobre el tema: Aragon y Portugal (2009), que utiliza un ejercicio de calibración y estimación por máxima verosimilitud en un modelo back-looking para la estimación de las preferencias, y Palma y Portugal (2011), que usa un modelo new-Keynesiano estándar con expectativas racionales para una economía cerrada y estimación por el filtro de Kalman y el método de la máxima verosimilitud. El modelo utilizado en este estudio considera que el Banco Central minimiza una función de pérdida, considerando el desvío de la inflación en relación a la meta, la estabilización del producto, la suavización de la tasa de intereses y, distintamente de los trabajos anteriores, la tasa de cambio. Los resultados permiten afirmar que la mayor preocupación de la autoridad monetaria en el periodo fue con la estabilización de la inflación, seguida por la suavización de la tasa de intereses, estabilización del producto y, por último, la estabilización de la tasa de cambio.

Palabras clave: política monetaria, preferencias del Banco Central de Brasil, estimación bayesiana, DSGE.

JEL: C11, E12, E52, E61

Abstract: The main objective of this paper is to estimate the preferences of the Central Bank of Brazil after the inflation targeting regime (January 2000 to June 2011), using a DSGE model with microeconomic foundations for a small open economy, based especially on the work of Kam, Lees and Liu (2009). In the Brazilian literature, there are two main works on the subject: Aragon and Portugal (2009), which uses a calibration exercise and maximum likelihood estimation in a backward-looking model for the estimation of preferences, and Palma and Portugal (2011), which uses a standard new-Keynesian model with rational expectations for a closed economy and estimation by the Kalman filter and maximum likelihood. The model used in this study considers that the Central Bank minimizes a loss function, taking into account the deviation of inflation from its target, output stabilization, the interest rate smoothing and, unlike the previous works, the exchange rate. The results allow us to affirm that the major concern of the monetary authority in the period was the stabilization of inflation, followed by interest rate smoothing, output stabilization and, finally, exchange rate stabilization.

Keywords: Monetary policy, Central Bank preferences, Bayesian estimation, DSGE.

JEL codes: C11, E12, E52, E6.
1. Introduction and Justification for the Study

One of the major developments in macroeconomics in the past few decades has been the adoption of the intertemporal utility maximization paradigm and its implementation in dynamic stochastic general equilibrium (DSGE) models, which are currently the predominant approach to macroeconomic analysis, not only among scholars, but increasingly among Central Banks worldwide.

In DSGE models, economic agents (consumers and firms) are treated as optimizing agents. Thus, families maximize utility conditional on some given budget constraint and firms maximize their profits with the applicable constraints. However, the Central Bank’s behavior is often described as a special case, i.e., by a monetary policy rule - the Taylor rule. Unlike other agents, mostly through DSGE models, the Central Bank does not solve its optimization problem.

The Taylor rule, the standard tool for assessing the behavior of Central Banks, empirically describes the monetary authority’s response to macroeconomic variables. Although it was proposed from a purely empirical perspective, the Taylor rule has a theoretical basis, being the solution to a restricted optimization problem, where the Central Bank minimizes a quadratic loss function. This way, the coefficients estimated in a reaction function are rather complex combinations of preference parameters (coefficients of the objective function) of the monetary authority and structural parameters of the model. Therefore, the coefficients found in the reaction function are reduced-form estimates and do not describe the structural characteristics of the monetary policy, and are then not useful for assessing issues concerned with the process of monetary policy formulation.

The aim of the present paper is to symmetrically deal with the monetary authority in a DSGE model, i.e., to assume that the Central Bank is also an optimizing agent and minimizes its loss function conditional on economic constraints and, based on this problem, to estimate the monetary authority’s preferences. Few works used a similar approach to the international case. Ilbas (2010b) and Ilbas (2010a) estimate FED’s preferences and the preferences for the euro zone, respectively, following the model proposed by Smets and Wouters (2003), under commitment. Kam, Lees and Liu (2009), on the other hand, estimate the central bank preferences for three of the major

---

small open economies that operate under the inflation targeting regime: Australia, Canada and New Zealand. The authors use a quadratic loss function and the model put forward by Monacelli (2005) as a constraint on the optimization problem, taking into account the discretionary case. In the Central Bank’s loss function, the following variables are used as arguments: deviation of inflation, of output, interest rate smoothing, and exchange rate. Remo and Vasícek (2009) do the same for the Central Bank of the Czech Republic, but they use commitment instead. In the loss function, the authors do not regard the exchange rate as argument.

Getting to know the Central Bank’s preferences is of utmost importance. Many inflation episodes, for instance, may arise from the monetary authority’s attempt to stabilize output above its natural rate. Moreover, the heavier the relative weight of output on the loss function, the larger the inflationary bias towards the economy. According to Castelnuovo and Surico (2003, p. 336), knowing the monetary policy preferences allows assessing its performance in a more accurate fashion, since it is possible to know whether the obtained result is that which was actually sought by the Central Bank or whether it represents just a random gain from favorable macroeconomic conditions.

In Brazil, Aragon and Portugal (2009) were the first authors to investigate the monetary authority’s preferences. In a calibration exercise of the loss function, the authors, using a backward-looking model, choose preference parameter values that minimize the deviation between the simulated optimal path and the actual path of the Selic rate. Given that in a calibration exercise, inference would not be possible, the optimal structure is employed to estimate preferences through the maximum likelihood method. The results indicate that the Central Bank of Brazil places a heavier weight on the inflation rate and that the concern with interest rate smoothing is deeper than with the output gap. Nevertheless, it should be noted that in the maximum likelihood estimation exercise the loss function parameters were not significant. This fact, as pointed out by the authors, might have occurred owing to the small sample size.

Palma and Portugal (2011) used a standard new-Keynesian model with forward-looking expectations proposed by Givens (2010) to estimate the monetary authority’s preferences in Brazil during the inflation targeting regime, taking into account a closed
Assuming rational expectations, it is necessary to make a key distinction about how agents’ expectations are dealt with in the optimization problem, i.e., by telling commitment and discretion apart. The main difference between these two possibilities lies in how agents’ expectations are treated in the Central Bank’s optimization problem. Equilibrium in both cases takes the form of a state-space model that can be estimated by maximum likelihood using the Kalman filter. The results show that the monetary authority places a heavier weight on inflation stabilization, followed by interest rate smoothing and by output stabilization. In addition, the results obtained by the authors indicate that a discretionary policy is more consistent with the data available for the analyzed period.

The present paper does innovate, offering a one-of-a-kind study for the Brazilian case, by using a DSGE model, which is more consistent with the optimization and rational expectations approach. The utilization of forward-looking expectations highlights the central role of expectations formation, especially under the inflation targeting regime, underscoring the importance of future events for the present time. Furthermore, the use of Bayesian methods in the estimation process is quite attractive due to the short duration of the sample period.

The aim of the present paper is to use the Bayesian approach to estimate the monetary authority’s preferences based on a DSGE model, where all structural equations result from optimal decisions made by private agents and policymakers in a discretionary context. The intention is to contribute to improving the insight into monetary policy conduct in Brazil under the inflation targeting regime, managing to remedy some shortcomings seen in previous works on the same topic. The Bayesian approach is quite attractive for this case, given that the sample for the inflation targeting regime is somewhat small.

The remainder of this paper is organized as follows. Section 2 introduces the methodology used, with a brief description of DSGE models and their estimation by Bayesian methods. Section 3 outlines the theoretical model used in the present paper, which is based chiefly on Kam, Lees and Liu (2009). Section 4 presents the data and the selection of priors, as well as the results. Section 5 concludes.

4 The authors tried to minimize this constraint by using a monetary conditions index (MCI) as monetary policy instrument, which is a weighted average between the interest rate and the exchange rate, in lieu of interest rates. However, the results did not vary significantly.
5 A discretionary environment was chosen, a priori, considering the results obtained by Palma and Portugal (2011).
2. Method

2.1. DSGE Models

In the 1960s and 1970s, the large-scale macroeconomic models (systems of equations) arose as an attractive tool for policymakers and economists. However, these models were harshly criticized on the grounds of their empirical, and mainly theoretical, foundations, since they were especially subject to Lucas (1976) critique, according to which, changes in economic policy modify agents’ expectations and, in turn, alter the parameters of economic models. The implications for the systems of equations models are catastrophic, as these models are believed to be of little value.

As an alternative, Kydland and Prescott (1982) proposed the first model that used a DSGE approach, or the real business cycle (RBC) approach to macroeconomic modeling. Nonetheless, total price flexibility was considered at first, implying that monetary authority’s actions do not influence real variables. Because of that, these models were initially unattractive for Central Banks and other organizations.

With the advent of nominal and real rigidities in the 1990s, besides imperfect competition in DSGE models, they eventually proved to be very useful in capturing important characteristics of macroeconomic time series, unlike their predecessor (the RBC models proposed by Kydland and Prescott). Aside from that, improvements in quantitative methods were key to arousing the interest in these models. Since then, these models have been constantly improved and, currently, DSGE models are the standard tool for macroeconometric analysis, and can be used for several purposes, such as estimation, prediction, comparison of models, identification of shocks and analysis of economic policies. The addition of expectations to these models renders them less vulnerable to Lucas critique than conventional models, where expectations are not used or are used in a limited way.

DSGE models are open to many criticisms, though. One of their limitations is that they develop strong hypotheses about the agents’ rationality. In addition, the heterogeneity of individuals is not taken into proper consideration.

In Brazil, improvements in DSGE models are particularly interesting given the adoption of the inflation targeting regime. Monetary policy is not randomly implemented and the Central Bank needs a vast array of models and tools to support its decisions, and DSGE models have taken on an increasingly important role.
2.2. Estimation of DSGE models

The estimation of DSGE models poses several challenges, since their parameters are highly nonlinear. Initially, these models were solved by calibration, a technique that basically consists in setting parameter values based on some prior knowledge. For formal estimation, one of the first approaches was the generalized method of moments, which includes endogenous and expectational variables, found in the relationships defined by the model. However, this strategy was not so robust in that it may be necessary to use quite big samples to have useful inferences.

Notwithstanding the numerous criticisms, Bayesian methods have become quite popular and are particularly suitable for the estimation of the models contemplated herein. Simply put, Bayesian statistic can be considered a mix of calibration and the maximum likelihood principle. Uncertainty, as well as previous knowledge about the model and its parameters, is described by prior probabilities. Comparison with the data via the likelihood function leads to the revision of these probabilities, yielding the posterior probabilities.

A DSGE model is basically a nonlinear system of equations in expectational differences, which can be written in the following general matrix form:

\[ E_t \{ f_\theta(y_{t+1}, y_t, y_{t-1}, \varepsilon_t, \varepsilon_{t+1}) \} = 0 \]  

(2.1)

where

\( \varepsilon_t \sim i.i.d. (0, \Omega_\varepsilon) \) is a random vector of size \( r \times 1 \) of structural shocks

\( E_t(\varepsilon_{t+1}) = 0 \)

\( E_t(\varepsilon_{t+1} \varepsilon_{t+1}') = \Omega_\varepsilon \)

\( y_t = \) vector of endogenous variables

As observed above, the model is stochastic, forward-looking and nonlinear. Regardless of the estimation method, it is necessary to solve this model first and to obtain its reduced form. To do that, a linear approximation to the model is performed at first and, through methods for the solution of rational expectations models, a solution is found to the linear system, expressed in terms of deviations from the steady state. Some

---

\(^6\) This section is widely based on Adjemian (2007).
of the solution methods frequently used are: Blanchard and Kahn, Sims, Klein, undetermined coefficients, among others.\(^7\)

Assuming there is a unique stable and invariant solution, it is given by an equation in nonlinear stochastic differences, where the endogenous variables are written as a function of their previous levels and of the contemporaneous structural shocks:

$$y_t = g_\theta(y_{t-1}, \varepsilon_t)$$  \hspace{1cm} (2.2)

\(g_\theta\) is a set of policy functions.

In general, it is not possible to obtain a closed solution to the model, and (local or global) approximation methods then have to be used. The Dynare software uses a local approximation around the deterministic steady state \((\bar{y}(\theta))\), i.e., the model is linearized around \(\bar{y}(\theta)\), such that \(f_\theta(\bar{y}, \bar{y}, \bar{y}, 0, 0) = 0\) and \(y^+ = g_\theta(y^+, 0)\).

Substituting (2.2) into (2.1) for \(y_t\) and \(y_{t+1}\):

$$E_t[f_\theta(g_\theta(g(y_{t-1}, \varepsilon_t), \varepsilon_{t+1}), g_\theta(y_{t-1}, \varepsilon_t), y_{t-1}, \varepsilon_{t+1}, \varepsilon_t)] = 0$$  \hspace{1cm} (2.3)

Each equation in (2.1) can then be approximated by the expected value of a Taylor expansion of its logarithm around the steady state (log-linearization). The equations in (2.1) and (2.2) can be approximated, using the following system (variables with a caret over them stand for percentage deviations of the original variables from their steady state):

$$E_t\{f_{y_{t+1}}\hat{y}_{t+1} + f_y\hat{y}_t + f_{y_{t-1}}\hat{y}_{t-1} + f_{\varepsilon_{t+1}}\varepsilon_{t+1} + f_{\varepsilon t}\varepsilon_t\} = 0$$  \hspace{1cm} (2.4)

$$\hat{y}_t = g_{y_{t-1}}\hat{y}_{t-1} + g_\varepsilon\varepsilon_t$$  \hspace{1cm} (2.5)

where

$$f_{y_{t+1}} = \frac{df_\theta}{dy_{t+1}}, \hspace{0.5cm} f_y = \frac{df_\theta}{dy_t}, \hspace{0.5cm} f_{y_{t-1}} = \frac{df_\theta}{dy_{t-1}}, \hspace{0.5cm} f_{\varepsilon_{t+1}} = \frac{df_\theta}{d\varepsilon_{t+1}}, \hspace{0.5cm} f_\varepsilon = \frac{df_\theta}{d\varepsilon_t},$$

$$g_{y_{t-1}} = \frac{dg_\theta}{dy_{t-1}} \quad \text{(feedback matrix\(^8\))} \quad g_\varepsilon = \frac{dg_\theta}{d\varepsilon_t} \quad \text{(feedforward matrix\(^9\))}$$

---

\(^7\) See details in Dejong and Dave (2007).

\(^8\) It represents the impact of endogenous variables on forward-looking variables.

\(^9\) It represents the impact of shocks on forward-looking variables.
The model in its linearized form can be solved with the help of the Dynare software in order to obtain its representation in its reduced state-space form. As a matter of fact, a series of complex algebraic procedures is needed.\textsuperscript{10} After finding the solution to the model in terms of its policy functions, one can write it in state-space form:

\[
\begin{align*}
y_t^* &= F \hat{y}_t + Gu_t & \text{measurement equation} \\
\hat{y}_t &= D \hat{y}_{t-1} + G \varepsilon_t & \text{state equation}
\end{align*}
\]

Maximum likelihood estimation requires that the likelihood function be constructed and assessed based on the structural parameters. This is complicated when the model includes unobservable state variables. In this case, the Kalman filter, for instance, allows making inferences about the unobservable state vector and assessing the joint likelihood function of observable endogenous variables, yielding consistent and asymptotically normal estimates for the parameters of interest.

If the DSGE model is stochastically singular (more observable variables than random shocks), maximum likelihood estimation often fails. When that occurs, two strategies can be considered: i) using at most as many observable variables as the number of structural shocks; ii) adding error terms in the state-space observation equation.

Nevertheless, empirical experience has demonstrated that it is too difficult to estimate a model by the maximum likelihood method. Usually, the function is flat in certain directions, giving rise to important identification problems. The solution came from Bayesian methods, where the identification problem is not a limiting factor.

The Bayesian statistic basically consists in treating the parameters as random variables. In the case of structural models, the use of Bayesian methods is a lot more attractive, since there is an interpretation for the parameters that are being estimated, facilitating the selection of priors.

The specification of priors for the parameters begins with the selection of the functional form that is more suitable for the distribution. For example, one can use the possible interval for the parameter values as reference. Thus, the inverse gamma distribution is used for parameters with only positive values, the beta distribution is reserved for parameters with values between 0 and 1 and normal distribution is used for

\textsuperscript{10} For a detailed description of these procedures, see Klein (2000) and Sims (2002).
unrestricted parameter values. The use of non-informative priors (uniform distribution) is also possible.

With the likelihood function and the specification of priors, it is possible to estimate the posterior distributions, which represent the probabilities associated with different parameter values after observation of the data. Basically, the posteriors are updates of beliefs, represented by the priors, based on additional information provided by variables in the sample. The determination of posteriors basically consists of the application of the widely known Bayes’ Theorem:

\[
p(\theta|y^*) = \frac{p(\theta, y^*)}{p(y^*)} = \frac{p(y^*|\theta)p(\theta)}{p(y^*)}
\]

where:
\(p(y^*|\theta)\) = likelihood function
\(p(\theta)\) = informative priors
\(p(y^*)\) = marginal density function of the sample.

Given that \(p(y^*)\) does not rely on the vector of parameters, it can be treated as a constant and the posterior can be written as

\[
p(\theta|y^*) \propto p(y^*|\theta)p(\theta) = K(\theta|y^*)
\]

where \(K(\theta|y^*)\) is known as the posterior kernel and is proportional to the posterior determined by factor \(p(y^*)\).

The determination of posteriors includes the calculation of quite nontrivial integrals and computationally intensive numerical methods are needed. The Markov Chain Monte Carlo (MCMC) methods allow for this calculation. The basic idea of this method is to construct a Markov chain with state spaces in parametric space \(\theta\), which is easy to simulate and whose equilibrium distribution is given exactly by the posterior distribution. The Metropolis-Hastings algorithm is one of the possibilities in this case. This algorithm is used herein and is summarized in what follows.

After defining a transition kernel, \(q(\theta, \beta)\), of the prior distribution, use it to produce candidates:

i) Begin with a value \(\theta^{(0)}\) and stage index \(j = 0\);
ii) Generate a point $\beta$ from the transition kernel

iii) Update $\theta^{(j)}$ into $\beta = \theta^{(j+1)}$ with probability given by $p = \min\left\{1, \frac{p(\beta)q(\theta^{(j)}, \beta)}{p(\theta^{(j)})q(\beta, \theta^{(j)})}\right\}$

iv) Keep $\theta^{(j)}$ with probability $1-p$

v) Repeat the procedure above until a stationary distribution is obtained.

Note that the computational cost of the implementation of DSGE models is relatively low. Using the Dynare software, the relevant issue focuses on the interpretation of results rather than on their mechanics.

3. Theoretical Model

The model used herein is based on the framework developed by Gali and Monacelli (2005), Monacelli (2005) and Justiniano and Preston (2010), which has been widely used for the analysis of fiscal and monetary policies by the central banks of small open economies (Australia, New Zealand, Canada, Czech Republic, Brazil, etc.). In the original model, monetary policy is described by an empirical Taylor rule. Following Ilbas (2010a, 2010b), and mainly Kam, Lees and Liu (2009), this hypothesis will be abandoned and we will assume that the monetary authority optimizes a quadratic loss function in a discretionary fashion, in accordance with the results obtained by Palma and Portugal (2011). This implies that the monetary authority reoptimizes the loss function in each period, taking the agents’ expectations as given.\(^{11}\)

The aggregate demand and supply curves are derived from the agents’ optimization problem (families and firms, respectively) with forward-looking expectations. The source of real rigidity is the consumption habit persistence and that of the nominal rigidity is the indexation to the previous inflation, as well as the hypothesis of monopolistic competition with sticky prices for domestic and importing firms.

\(^{11}\) Note that the inflation targeting regime is an instrument that increases transparency, communication and coherence of the monetary policy, not necessarily consisting of a conventional strict commitment system (Bernanke and Mishkin, 1997). In fact, this regime is compatible with the discretionary behavior of the monetary policy. According to Mendonça (2001), owing to the transparency of the inflation targeting regime, it is possible to use discretionary policies without loss of credibility of the monetary authority.
Moreover, one should consider the imperfect exchange rate pass-through. Finally, we add the Central Bank’s optimization problem instead of the Taylor rules to describe the behavior of the monetary policy. The model is therefore made up of four agents: families, domestic firms, importing firms and the monetary authority.

3.1. Families

In the model, there exists a continuum of identical families that live infinitely, indexed by \( i \in (0,1) \). The overall population equals one. The utility function is given by:

\[
U(C_t, H_t, N_t) = \frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}
\]  

(3.1)

where \( C_t \) is a goods consumption index, \( H_t \) represents external habit formation, given by an autoregressive process of order 1, with parameter \( h \), and \( N_t \) are the working hours. The inverse elasticity of intertemporal substitution is given by parameter \( \sigma > 0 \), and the inverse of the elasticity of labor supply is given by \( \varphi > 0 \).

The goods consumption index, \( C_t \), is a combination of a continuum of domestic goods, \( C_{H,t}(i) \) and of imported goods, \( C_{F,t}(j) \), given by the CES function:

\[
C_t = \left[ (1 - \alpha)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{\eta-1}{\eta}}C_{H,t} \right]^{\frac{\eta}{\eta-1}},
\]  

(3.2)

where

\[
C_{H,t} = \left[ \int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}
\]  

and

\[
C_{F,t} = \left[ \int_0^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}
\]

The elasticity of substitution between domestic and imported goods is given by parameter \( \eta > 0 \), and the elasticity of substitution across goods within each category\(^{13} \) (domestic and imported goods) is given by \( \varepsilon > 0 \).

\(^{12}\) For further reading on this topic, see, for instance, Goldfajn and Werlang (2000). In short, with the imperfect exchange rate pass-through, importers do not immediately adjust the domestic price of imports in response to exchange rate fluctuations.

\(^{13}\) Note that within each category (domestic and imported goods), there are different products.
Solving the optimization problem of the families (utility maximization given the budget constraint), we obtain the following optimal demand functions:

\[ C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \]  
(3.3)

\[ C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \]  
(3.4)

where \( P_{H,t} \) and \( P_{F,t} \) are the aggregate price levels for the domestic economy and for the foreign economy, given respectively by:

\[ P_{H,t} = \left( \int_0^1 P_{H,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \]  
(3.5)

\[ P_{F,t} = \left( \int_0^1 P_{F,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \]  
(3.6)

The consumer price index is obtained by substituting the demand functions in the consumption index, \( C_t \):

\[ P_t = \left[ (1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \]  
(3.7)

### 3.2. Firms

#### 3.2.1 Domestic goods firms

Domestic goods operate with a linear production function given by \( Y_{H,t}(i) = \epsilon_{a,t} N_t(i) \), where \( \epsilon_{a,t} \) is an exogenous domestic technology shock, which follows an AR(1) process, \( Y_{H,t}(i) \) is the production of the ith firm, \( N_t(i) \) is the amount of hired labor.

Firms are monopolistically competitive, which introduces nominal rigidity in the model. In each period, a fraction \( \theta F \in [0,1] \) of the firms does not reoptimize their prices. Instead, these firms change their prices by indexing them to the previous inflation. The optimizing firms, however, solve their optimization problem, which is given by:

\[
\max_{P_{H,t}(i)} E_t \sum_{s=0}^{\infty} Q_{t,t+s} \theta_H^s Y_{H,t+s}(i) \left[ P_{H,t}(i) \left( \frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} - P_{H,t+s} \delta H_{t+s} e^{\epsilon_{H,t+s}} \right]
\]

Subject to the constrained demand of the ith firm:
Parameter $\varepsilon > 1$ is the elasticity of substitution between produced goods and $MC_{H,t}$ is the real marginal cost at time $t$, given by:

$$MC_{H,t} = \frac{W_t}{\varepsilon a_t P_{H,t}}$$

The structural shock to the marginal cost is given by $\varepsilon_{H,t} \sim i.i.d. (0, \sigma_H)$ and $\delta_H \in [0,1]$ is the level of inflation inertia.

The log-linearized first-order conditions of this problem give rise to the Phillips curve for domestic inflation, given by:

$$\pi_{H,t} - \delta_H \pi_{H,t-1} = \beta (E_t \pi_{H,t+1} - \delta_H \pi_{H,t}) + \lambda_H (mc_{H,t} + \varepsilon_{H,t})$$  
(3.9)

where

$$\lambda_H = \frac{(1-\beta H)(1-\theta H)}{\theta H}$$

$$mc_{H,t} = \varphi y_t - (1 + \varphi)\varepsilon a_t + \alpha s_t + \frac{\sigma}{1 - \eta} (y_t^* - hy_{t-1}^*) + q_t + \varepsilon_{c,t}$$

### 3.2.2. Importing firms

The basic idea is the same as for domestic firms and thus this section will be quite short. The model considers the monopolistic competition between importers and price setting à la Calvo. Furthermore, a gap is assumed between the price of imported goods denominated in domestic currency and the domestic price of imported goods, which can be explained by the fact that importing firms purchase imported goods at globally competitive prices. However, in the domestic economy, these firms are monopolistically competitive, redistributing these goods. This gap is given in log-linear terms by:

$$\psi_{F,t} = e_t + p_t^* - p_{F,t}$$  
(3.10)

The solution to the optimization problem of importing firms (as in the previous section) leads to the Phillips curve for imported goods inflation, given by:
\[
\pi_{F,t} = \beta \left( E_t \pi_{F,t+1} - \delta_F \pi_{F,t} \right) + \delta_F \pi_{F,t-1} + \lambda_F \left( \psi_{F,t} + \epsilon_{F,t} \right) \quad (3.11)
\]

where
\[
\lambda_F = \frac{(1-\beta_F)(1-\theta_F)}{\theta_F} e \psi_{F,t} = m c_{F,t}.
\]

### 3.3. Terms of trade, real exchange rate and equilibrium

In equilibrium, domestic output is equal to the total domestic demand (domestic and foreign demand) for produced goods, which in log-linear terms yields:

\[
y_t = c_{H,t} + c_{H,t}^* \quad (3.12)
\]

Performing the necessary substitutions, we get:

\[
y_t = (2 - \alpha) \alpha s_t + (1 - \alpha) c_t + \alpha \eta \psi_{F,t} + \alpha y_t^* \quad (3.13)
\]

It is also possible to derive a relationship between terms of trade \((s_t)\), the real exchange rate \((q_t)\) and the gap in the law of one price \((\psi_{F,t})\), given by:\(^{14}\)

\[
q_t = \psi_{F,t} - (1 - \alpha) s_t \quad (3.14)
\]

In what follows, we present the linearized version of the model, equation for equation.\(^{15}\)

### 3.4. The log-linearized model

The equations for the log-linearized model (i.e., the log-linear approximation to the first-order conditions and the constraints that describe the equilibrium of the economy) are presented below. Detailed information on how to obtain these equations can be found in Kam, Lees and Liu (2009). Note that terms of trade shocks, technology

---

\(^{14}\) For further details on the derivation of this relationship, see Kam, Lees and Liu (2009).

\(^{15}\) The description of parameters is given in tables 2 and 3.
shocks, and real interest rate parity shocks are, for simplicity, treated as exogenous stochastic processes.

**Consumption Euler equation**

\[ c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - \frac{1-h}{\sigma}(r_t - E_t\pi_{t+1}) \]  

(3.15)

**Domestic goods inflation**

\[ \pi_{H,t} = \beta E_t(\pi_{H,t+1} - \delta_H\pi_{H,t}) + \delta_H\pi_{H,t-1} + \lambda_H\left[\varphi y_t - (1 + \varphi)\varepsilon_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1})\right] + \lambda_H\varepsilon_{H,t} \]  

(3.16)

**Imported goods inflation**

\[ \pi_{F,t} = \beta E_t(\pi_{F,t+1} - \delta_F\pi_{F,t}) + \delta_F\pi_{F,t-1} + \lambda_F[q_t - (1 - \alpha)s_t] + \lambda_F\varepsilon_{F,t} \]  

(3.17)

**Real interest rate parity condition**

\[ E_t(q_{t+1} - q_t) = (r_t - E_t\pi_{t+1}) - (\pi_{t+1} - E_t\pi_{t+1}) + \varepsilon_{q,t} \]  

(3.18)

**Terms of trade equation (identity)**

\[ s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t} + \varepsilon_{s,t} \]  

(3.19)

**Goods market equilibrium condition**

\[ y_t = (1 - \alpha)c_t + \alpha\eta q_t + \alpha\eta s_t + \alpha y^*_t \]  

(3.20)

**General inflation**

\[ \pi_t = (1 - \alpha)\pi_{H,t} + \alpha\pi_{F,t} \]  

(3.21)

The *exogenous stochastic processes* for the terms of trade shocks, technology shocks and real interest rate parity shocks are written as:

\[ \varepsilon_{j,t} = \rho_j\varepsilon_{j,t-1} + \nu_{j,t} \]  

(3.22)

with \( \rho_j \in (0,1) \) and \( \nu_j \sim i.i.d. \left(0, \sigma_j^2\right) \), for \( j = s, a, q \).
$$\begin{pmatrix}
\pi_t^* \\
y_t^* \\
r_t^*
\end{pmatrix} = 
\begin{pmatrix}
ap_1 & 0 & 0 \\
0 & b_2 & 0 \\
0 & 0 & c_3
\end{pmatrix}
\begin{pmatrix}
\pi_{t-1}^* \\
y_{t-1}^* \\
r_{t-1}^*
\end{pmatrix} + 
\begin{pmatrix}
\sigma_{\pi^*} & 0 & 0 \\
0 & \sigma_{y^*} & 0 \\
0 & 0 & \sigma_{r^*}
\end{pmatrix}
\begin{pmatrix}
\nu_{\pi^*,t} \\
\nu_{y^*,t} \\
\nu_{r^*,t}
\end{pmatrix} \quad (3.23)
$$

where
$$\begin{pmatrix}
\nu_{\pi^*,t} \\
\nu_{y^*,t} \\
\nu_{r^*,t}
\end{pmatrix} \sim N(0, I_3).$$

### 3.5 Monetary Authority: Central Bank Preferences

Optimal monetary policy, which makes a distinction between the model presented above and that proposed by Gali and Monacelli (2005), will be outlined in what follows. The intertemporal loss function of a period of the Central Bank is given by:

$$L(\tilde{\pi}_t, y_t, q_t, r_t - r_{t-1}) = \frac{1}{2} \left[ \tilde{\pi}_t^2 + \mu_y y_t^2 + \mu_q q_t^2 + \mu_r (r_t - r_{t-1})^2 + \varepsilon_t^2 \right] \quad (3.24)$$

As can be seen above, the weight allocated to the inflation target is normalized at unity and, therefore, the weights of the other variables ($\mu_y, \mu_q, \mu_r \in [0, \infty]$) will be regarded as relative to that of the inflation target. As the inflation target is fixed in time, it will be normalized at zero, since all variables in the model are expressed as deviations from their mean. The last term of the loss function, $\Delta r_t = (r_t - r_{t-1})$, is justifiable by the fact that the monetary authority also worries about the financial stability or for considering the inertial behavior of the policy instrument. In addition, we allow for a shock to this variable, given by $\varepsilon_t \sim N(0, \sigma^2)$, which allows capturing the imperfect ability of the Central Bank to control the nominal interest rate. The inclusion of the exchange rate in the Central Bank’s loss function allows us to answer whether this variable is taken into consideration in the monetary authority’s optimization problem.

The monetary authority’s goal is to minimize the loss function, subject to structural economic equations (3.15)-(3.23), under discretion. The solution to the problem is found using the algorithm proposed by Dennis (2004). In sum, the concept

---

16 This quadratic function combined with linear restrictions produces linear decision rules. Moreover, it may represent a second-order approximation to the utility function of the representative agent.
of Markov perfect equilibrium is used, where the Central Bank nowadays (i.e., its contemporaneous decisions) is considered a Stackelberg leader and the private agents and the future Central Bank’s actions are regarded as Stackelberg followers.17

4. Estimation and Results

4.1. Data and selection of prior distributions

The model presented above will be estimated by Bayesian methods, as described in section 2. The data used are log-linearized quarterly series of the following variables, for the period after the inflation targeting regime (January 2000 – June 2011), totaling 46 observations:

- Imported goods inflation denominated in domestic currency, $\pi_{F,t}$;
- Real domestic exchange rate: RS/US$ (Ptax sale value at the end of the period), $q_t$;
- Final household consumption, $c_t$;
- Terms of trade - FUNCEX (exports and imports), $s_t$;
- Real domestic GDP – seasonally adjusted index, $y_t$;
- Domestic inflation: IPCA index, $\pi_t$;
- Nominal interest rate: annualized Selic rate, $r_t$;
- U.S. inflation, $\pi^*_t$;
- U.S. output, $y^*_t$;
- U.S. interest rate, $r^*_t$.

The series will be expressed as deviations from the sample mean and are available at www.ipeadata.gov.br, on the website of the Central Bank of Brazil at www.bcb.gov.br and on the website of IBGE at www.ibge.gov.br. The variables were seasonally adjusted and the trend was removed using the Hodrick-Prescott (HP) filter.

Table 4.1 shows the prior distribution of the parameters to be estimated. The selection of priors takes into account mainly the interval of variation of each parameter. Conventionally, we use the beta distribution for parameters that are on the interval [0,1], the inverse gamma distribution for those on the interval [0, ∞) and the gamma distribution for the remaining cases.

17 The solution to the problem is briefly shown in the Appendix.
Table 4.1. Prior Distribution of the Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Prior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Intertemporal discount rate</td>
<td>0.99*</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Level of economic openness (share of imports in domestic consumption)</td>
<td>0.45*</td>
</tr>
<tr>
<td>$h$</td>
<td>Habit persistence parameter</td>
<td>Beta (0.7, 0.1)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Inverse of the elasticity of substitution (coefficient of relative risk aversion)</td>
<td>Gamma (0.2, 0.2)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse of the elasticity of labor supply</td>
<td>Gamma (2, 0.35)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution between domestic and imported goods</td>
<td>Gamma (0.6, 0.25)</td>
</tr>
<tr>
<td>$\delta_H$</td>
<td>Backward-looking parameter of the price of domestic goods</td>
<td>Beta (0.7, 0.2)</td>
</tr>
<tr>
<td>$\delta_F$</td>
<td>Backward-looking parameter of the price of imported goods</td>
<td>Beta (0.7, 0.2)</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>Fraction of non-optimizing producers in the domestic economy</td>
<td>Beta (0.5, 0.2)</td>
</tr>
<tr>
<td>$\theta_F$</td>
<td>Fraction of non-optimizing importers.</td>
<td>Beta (0.5, 0.2)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>Parameter AR(1) of foreign inflation</td>
<td>Beta (0.5, 0.1)</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Parameter AR(1) of external output</td>
<td>Beta (0.5, 0.1)</td>
</tr>
<tr>
<td>$c_3$</td>
<td>Parameter AR(1) of foreign interest rate</td>
<td>Beta (0.5, 0.1)</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Technological inertia</td>
<td>Beta (0.8, 0.1)</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>Inertial effect of exchange rate shock</td>
<td>Beta (0.8, 0.1)</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Inertial effect of terms of trade shock</td>
<td>Beta (0.8, 0.1)</td>
</tr>
<tr>
<td>$\mu_q$</td>
<td>Preference for exchange rate stabilization</td>
<td>Gamma (0.5, 0.09)</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>Preference for output stabilization</td>
<td>Gamma (0.5, 0.09)</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>Preference for interest rate smoothing</td>
<td>Gamma (0.5, 0.09)</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>Standard deviation of the “cost-push” shock to the domestic economy</td>
<td>Inverse gamma (0.15, 2)</td>
</tr>
<tr>
<td>$\sigma_F$</td>
<td>Standard deviation of the “cost-push” shock to the foreign economy</td>
<td>Inverse gamma (0.15, 2)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Standard deviation of the technology shock</td>
<td>Inverse gamma (0.15, 2)</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>Standard deviation of the terms of trade</td>
<td>Inverse gamma (0.15, 2)</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Standard deviation of the terms of trade</td>
<td>Inverse gamma (0.15, 2)</td>
</tr>
<tr>
<td>$\sigma_{r^*}$</td>
<td>Standard deviation of the foreign inflation rate</td>
<td>Inverse gamma (0.1, 2)</td>
</tr>
<tr>
<td>$\sigma_{y^*}$</td>
<td>Standard deviation of the external output</td>
<td>Inverse gamma (0.15, 2)</td>
</tr>
<tr>
<td>$\sigma_{r^*}$</td>
<td>Standard deviation of the foreign interest rate</td>
<td>Inverse gamma (0.15, 2)</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>Standard deviation of the domestic interest rate</td>
<td>Inverse gamma (0.15, 2)</td>
</tr>
</tbody>
</table>

Note: The mean and standard deviation of the distributions are respectively shown in brackets.
* Denotes calibrated parameters.
4.2 Results

The model was estimated with Matlab, using Bayesian techniques through the Metropolis-Hastings algorithm and the Kalman filter,\(^\text{18}\) as described in section 2. The results are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Posterior Mean</th>
<th>95% CI</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h )</td>
<td>0.89</td>
<td>[0.87; 0.90]</td>
<td>0.01</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>1.09</td>
<td>[1.08; 1.10]</td>
<td>0.01</td>
</tr>
<tr>
<td>( \phi )</td>
<td>1.71</td>
<td>[1.62; 1.80]</td>
<td>0.05</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.13</td>
<td>[0.09; 0.18]</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Firms and Exogenous Processes</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_H )</td>
<td>0.31</td>
<td>[0.31; 0.32]</td>
<td>0.00</td>
</tr>
<tr>
<td>( \delta_F )</td>
<td>0.07</td>
<td>[0.06; 0.08]</td>
<td>0.01</td>
</tr>
<tr>
<td>( \theta_H )</td>
<td>0.66</td>
<td>[0.64; 0.69]</td>
<td>0.02</td>
</tr>
<tr>
<td>( \theta_F )</td>
<td>0.87</td>
<td>[0.82; 0.92]</td>
<td>0.03</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.88</td>
<td>[0.86; 0.89]</td>
<td>0.01</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.83</td>
<td>[0.78; 0.89]</td>
<td>0.03</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>1.04</td>
<td>[1.02; 1.07]</td>
<td>0.02</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>0.77</td>
<td>[0.76; 0.78]</td>
<td>0.00</td>
</tr>
<tr>
<td>( \rho_q )</td>
<td>0.45</td>
<td>[0.45; 0.48]</td>
<td>0.01</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>0.85</td>
<td>[0.83; 0.87]</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Monetary Policy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu_q )</td>
<td>0.19</td>
<td>[0.17; 0.21]</td>
<td>0.01</td>
</tr>
<tr>
<td>( \mu_y )</td>
<td>0.51</td>
<td>[0.50; 0.51]</td>
<td>0.00</td>
</tr>
<tr>
<td>( \mu_T )</td>
<td>0.63</td>
<td>[0.61; 0.64]</td>
<td>0.01</td>
</tr>
<tr>
<td>Absolute weights:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu_D = 0.4292, \mu_q = 0.0815, \mu_y = 0.2189, \mu_T = 0.2704 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Standard deviations of shocks</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_H )</td>
<td>0.84</td>
<td>[0.81; 0.87]</td>
<td>0.02</td>
</tr>
<tr>
<td>( \sigma_F )</td>
<td>3.67</td>
<td>[3.60; 3.71]</td>
<td>0.04</td>
</tr>
<tr>
<td>( \sigma_a )</td>
<td>7.78</td>
<td>[7.72; 7.82]</td>
<td>0.03</td>
</tr>
<tr>
<td>( \sigma_q )</td>
<td>2.64</td>
<td>[2.51; 2.79]</td>
<td>0.10</td>
</tr>
<tr>
<td>( \sigma_s )</td>
<td>4.91</td>
<td>[4.85; 4.97]</td>
<td>0.04</td>
</tr>
<tr>
<td>( \sigma_{\pi}^* )</td>
<td>0.80</td>
<td>[0.73; 0.87]</td>
<td>0.05</td>
</tr>
<tr>
<td>( \sigma_{y}^* )</td>
<td>0.64</td>
<td>[0.56; 0.71]</td>
<td>0.04</td>
</tr>
<tr>
<td>( \sigma_{r}^* )</td>
<td>0.51</td>
<td>[0.47; 0.54]</td>
<td>0.02</td>
</tr>
<tr>
<td>( \sigma_T )</td>
<td>3.84</td>
<td>[3.64; 3.98]</td>
<td>0.12</td>
</tr>
</tbody>
</table>

\(^{18}\) We used the codes kindly granted by Timothy Kam as reference.
For the sake of comparison of the results obtained for the preference parameters, we show the results of previous studies for Brazil and for other countries that adopt the inflation targeting regime, in tables 4.2 and 4.3, respectively.

Table 4.2. Comparison with previous studies

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2000 to 2007</td>
<td>2000-1 to 2010-4</td>
<td>2000-1 to 2011-4</td>
</tr>
<tr>
<td>$\mu_\pi$</td>
<td>0.727</td>
<td>0.8264</td>
<td>0.4292</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>0.073</td>
<td>0.0083</td>
<td>0.2189</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>0.2</td>
<td>0.1653</td>
<td>0.2704</td>
</tr>
<tr>
<td>$\mu_q$</td>
<td>--------</td>
<td>--------</td>
<td>0.0815</td>
</tr>
</tbody>
</table>

Table 4.3. Comparison with the international literature: “Small Inflation Targeters”

<table>
<thead>
<tr>
<th>Preference parameters</th>
<th>Canada</th>
<th>Australia</th>
<th>New Zealand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1990-1 to 2005-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_\pi$</td>
<td>0.4953</td>
<td>0.4931</td>
<td>0.4697</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>0.0778</td>
<td>0.2032</td>
<td>0.1282</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>0.4235</td>
<td>0.3013</td>
<td>0.3992</td>
</tr>
<tr>
<td>$\mu_q$</td>
<td>0.0035</td>
<td>0.0025</td>
<td>0.0028</td>
</tr>
</tbody>
</table>

Source: Built based on the results obtained by Kam, Lees and Liu (2009)

As shown by the results above, the Central Bank attaches greater value to the stabilization of inflation around its target (reference value, with weight equal to 1), followed by interest rate smoothing (0.63), by output stabilization (0.51) and, finally, by exchange rate stabilization (0.19). The absolute weights are shown in table 4.1. Table 4.2 makes a comparison with previous results found in the literature. The order of parameter values does not change from one study to the other, but the magnitude is quite different. This is not a surprising result because we use a DSGE model, unlike other studies, which were conventional econometric models. In addition, the model used in this paper takes into account a small open economy, contrary to the previous studies. Palma and Portugal (2011) used a closed economy model and Aragon and Portugal (2009) employed only a random walk to describe the exchange rate behavior.

The results obtained herein show that the monetary authority attaches great weight to inflation stabilization, but a lower weight than those observed earlier. Moreover, note that there is a very deep concern with interest rate smoothing, with similar results to those published in the international literature. Kam, Lees and Liu
(2009) found similar results for other “small open economies” that adopt the inflation targeting regime: Australia, Canada and New Zealand. Also noteworthy is the fact that the weight given to output stabilization is significantly heavier than that attached in previous studies. This can be accounted for by the larger sample size used. In recent times, the Central Bank has apparently given a larger weight to output gap, which may be helping to increase the weight of this variable in the loss function.

Conversely, the positive weight for the exchange rate smoothing parameter can be seen as an attempt to reduce the volatility of inflation in the short run, i.e., the weight of exchange in the reaction function is indirectly associated with inflation control.

The habit formation parameter, estimated at 0.89, shows the relevance of habit formation to Brazil. Silveira (2008) reports values for the first-order habit persistence between 0.55 and 0.81. Silva and Portugal (2010) obtain a value of 0.9562, and a value of 0.74 for the SAMBA model for Brazil. According to Cavalcanti and Vereda (2011, p. 16), there are few references about persistence values for Brazil, since most recent works do not take this characteristic into consideration. Nonetheless, the value found herein is consistent with the recent literature.

The intertemporal elasticity of substitution in consumption (estimated at 0.92, $\sigma = 1.09$) is relatively higher than the values reported in the literature. Silveira (2008) found a value of 0.48, and concluded that the aggregate demand responds to changes in the real interest rate, representing the conventional and effective monetary policy transmission mechanism in the Brazilian economy. Silva and Portugal (2010) also find lower values for the elasticity of substitution (1.2234, which implies an elasticity of 0.8174). On the other hand, the SAMBA model estimates this parameter at 0.77 ($\sigma = 1.30$). The result indicates that the tendency towards smoothing consumption in Brazil is quite strong and larger than in the euro zone (Silva and Portugal, 2010). As stated by Silveira (2008, p. 350), due to the large variability of the results, it is not possible to draw a definitive conclusion for this parameter. Specifically in our case, the posterior distribution is not very different from the prior one, probably indicating poor identification of this parameter.19

The elasticity of labor supply was estimated at 0.58 ($\phi = 1.71$). Silveira (2008) found a relatively larger value, 0.77 ($\phi = 1.30$), as well as Gouvea et al. (2011), who obtained 0.338 for this parameter (which implies an elasticity of 2.96). Yet, as

---

19 As pointed out by Castro et al. (2011), the identification of this parameter is often cumbersome.
suggested by the international literature, this value should be lower, as the one found herein. Silva and Portugal (2010) obtained a value of $\phi = 1.8128$ (elasticity of 0.55), quite close to the one estimated in this paper. This parameter can be interpreted as the percentage change in labor supply given by a percentage change in real wage. The low value obtained here compared to other studies could refer to the specificity of rigidity in the Brazilian labor market.

The elasticity of substitution between domestic and imported goods was estimated at 0.13, which indicates poor chances of substitution among these goods. The estimates for price stickiness parameters (Calvo) are consistent with the referenced literature. For the Brazilian economy, this elasticity was estimated at 0.66 and for the U.S. economy, at 0.87. The estimates for the backward-looking components of the Phillips curve were extremely low. The persistence parameters of exogenous processes are very high and most of them match those estimates described in the literature.

5. Conclusion

The major aim of this paper was to estimate the Central Bank preferences using a DSGE model for a small open economy, based chiefly on the work of Kam, Lees and Liu (2009). This topic has been investigated only recently for the Brazilian case and, to the best of our knowledge, only two studies estimated the preferences of the Central Bank of Brazil: Aragon and Portugal (2009) and Palma and Portugal (2011). The present study innovates in relation to both by considering a DSGE model and extending the work of Palma and Portugal (2011), for a small open economy. Additionally, the use of the exchange rate variable in the Central Bank’s loss function is a novelty for the Brazilian case.

The model was estimated using quarterly data (in order to minimize measurement errors and the number of lags in the model) in the period that followed the inflation targeting regime (January 2000 to June 2011). Most of the results obtained for the structural economic parameters are consistent with the main previous studies that use DSGE models to assess issues related to the Brazilian economy.

As to the preference parameters, it is possible to assert that the Central Bank attaches heavier weight to the stabilization of inflation around its target, but that it is also interested in interest rate smoothing, output stabilization and exchange rate stabilization, in this strict order.
The study sought to improve the estimation of Central Bank preferences. To achieve that, a DSGE model with microeconomic foundations for a small open economy was used, consistent with the optimization and rational expectations approaches. Unlike most DSGE models, and an innovation for the Brazilian case, the monetary authority was regarded also as an optimizing agent. Contrary to other agents (families and firms, for example), in most of the studies using DSGE models, the Central Bank does not solve its optimization problem, and its behavior is described through the Taylor rule.

Some extensions to this study can and should be conducted in the future in order to improve and shed further light upon this important topic. One of them consists in including some elements of the SAMBA model, notably an equation for the inflation target that best describes the behavior of the Brazilian economy. In addition, as Palma and Portugal (2011) did, a version of the commitment model could be estimated so as to check which of the two cases (commitments × discretion) best suits the Brazilian case.
REFERENCES


CAVALCANTI, M.A.F.H.; VEREDA, L. Propriedades dinâmicas de um modelo DSGE com parametrizações alternativas para o Brasil. IPEA. Texto para Discussão 1588, março 2011.


APPENDIX

A.1. Central Bank optimization problem – Discretion

In a nutshell, we will describe the solution to the model for the discretionary case. This section is based on Dennis (2004) and Kam, Lees and Liu (2009). The Central Bank’s problem can be written as:

$$\min_{(x_t)^\infty} L(t, \infty) = E_t \sum_{j=0}^{\infty} \beta^j [y_t^i W y_t + x_t^i Q x_t]$$

s.a.: $A_0 Y_t = A_1 Y_{t-1} + A_2 E_t Y_{t+1} + A_3 x_t + A_4 E_t x_{t+1} + A_5 w_t$

This problem is solved by proposing a solution (law of motion) as shown next and by finding the fixed points of $H_1, H_2, F_1$ and $F_2$:

$$Y_t = H_1 Y_{t-1} + H_2 v_t$$

$$X_t = F_1 Y_{t-1} + F_2 v_t$$

Note that the Markov perfect equilibrium is temporally consistent, i.e., there are no incentives for current and future deviations by the Central Bank.

A.2. Model estimated under commitment

In the present paper, we followed the results found by Palma and Portugal (2011) and we estimated the model under discretion. It is interesting to check whether the results obtained would significantly change by estimating the model under commitment. The results in table A.2. refer to the same model contemplated in this paper, but now estimated under commitment. As observed, the results do not change significantly. Here, we only note that the exchange rate deviation has a slightly heavier weight than the output deviation.
Table A.2. Model estimated under commitment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Posterior Mean</th>
<th>95%CI</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>0.87</td>
<td>[0.87; 0.87]</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.10</td>
<td>[1.10; 1.11]</td>
<td>0.00</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.40</td>
<td>[1.36; 1.45]</td>
<td>0.03</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.28</td>
<td>[0.27; 0.30]</td>
<td>0.01</td>
</tr>
<tr>
<td>Firms and Exogenous Processes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_H$</td>
<td>0.05</td>
<td>[0.05; 0.06]</td>
<td>0.00</td>
</tr>
<tr>
<td>$\delta_F$</td>
<td>0.05</td>
<td>[0.04; 0.06]</td>
<td>0.00</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>0.73</td>
<td>[0.73; 0.74]</td>
<td>0.00</td>
</tr>
<tr>
<td>$\theta_F$</td>
<td>0.66</td>
<td>[0.66; 0.67]</td>
<td>0.00</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.69</td>
<td>[0.66; 0.72]</td>
<td>0.02</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.69</td>
<td>[0.68; 0.71]</td>
<td>0.01</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.96</td>
<td>[0.95; 0.97]</td>
<td>0.01</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.78</td>
<td>[0.77; 0.79]</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>0.44</td>
<td>[0.44; 0.44]</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>0.78</td>
<td>[0.78; 0.79]</td>
<td>0.00</td>
</tr>
<tr>
<td>Monetary Policy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_q$</td>
<td>0.50</td>
<td>[0.49; 0.50]</td>
<td>0.00</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>0.49</td>
<td>[0.49; 0.49]</td>
<td>0.00</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>0.65</td>
<td>[0.64; 0.65]</td>
<td>0.00</td>
</tr>
<tr>
<td>Absolute Weights: $\mu_\pi = 0.3788, \mu_q = 0.1894, \mu_y = 0.1856, \mu_r = 0.2462$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation of shocks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>0.86</td>
<td>[0.81; 0.87]</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_F$</td>
<td>3.00</td>
<td>[2.97; 3.01]</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>7.78</td>
<td>[7.77; 7.79]</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>1.99</td>
<td>[1.96; 2.04]</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>5.01</td>
<td>[4.98; 5.04]</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma_{\pi^*}$</td>
<td>0.55</td>
<td>[0.53; 0.57]</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_{y^*}$</td>
<td>0.52</td>
<td>[0.51; 0.53]</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_{r^*}$</td>
<td>0.36</td>
<td>[0.34; 0.37]</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>3.84</td>
<td>[3.64; 3.98]</td>
<td>0.12</td>
</tr>
</tbody>
</table>