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Competition for Local Public Services with Learning-by-doing and Transferability*

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Abstract

Many local governments allow competition between public and private firms for provision of local public services in order to reduce procurement cost. Competition is usually introduced through competitive tendering for concession contracts. We show that in a symmetric competition between public and private firms with learning-by-doing, private firm's ability to transfer learning among concessions may reduce consumer's welfare. The model provides testable implications which are consistent with the empirical evidence: little competition for concessions, retail prices higher under private operation than under public one, and subsidies and retail prices to service providers increased over time. In addition, consumers' gains from switching to private ownership are higher in industries where private firms have low-ability to transfer learning among different concessions.

JEL: D44; H57; H70; H87.

Keywords: Sequential Auction, Public versus Private Firms, Learning-by-doing, Transferability of Learning.

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1 Introduction

In most of European countries, the local public authorities are in charge of the organization of local public services, which includes the choice of the service provider and price regulation (see, Szymanski (1996), Gomez-Lobo and Szymanski (2001), and Chong and Huet (2009)). Since the 1980s, several local governments have allowed competition between public and private firms for provision of public services. Many public services, like potable water, public transportation, refuse collection services, and street repairs, were previously provided by local public firms ran by civil servants, turned out to be managed by private firms. One of the main arguments in favor of competition for those services was the potential gains in efficiency, i.e., reduction in cost and prices (see, e.g. Megginson and Netter, 2001; Martimort, De Donder and De Villemeur, 2005).

The empirical evidence from certain industries shows that this strategy does not seem to be very successful. For instance, GEA-ENGREF (2002), a recent report on contracts of water concession in France, documents that in 30% of auctions for concession, there is only one bidder. Moreover, only 12% of auctions result in the incumbent being replaced. These facts provide evidence that there is little competition for water concession in France. In addition, Bontemps, Martimort, Roucolle and Thomas (2009), analyzing the regulated price of potable water in France, show that water prices in cities with private ownership are on average higher than in cities with public ownership. In the transportation sector, Gagnepain, Ivaldi and Martimort (2009), analyzing the government subsidies to providers of local public transportation in France, show that subsidies to public and private firms have been increasing over time, which may be associated to a reduction in consumer welfare.

We argue that the existence of learning-by-doing in the provision of public services and the private firms' ability to transfer learning among concessions in markets with low degree of competition, may explain the empirical evidence.

Learning-by-doing allows an incumbent firm to reduce its cost through time, becoming more efficient. This gives to the incumbent an advantage over the entrant in future competitions for public services. In such environment, the incumbent has always low production cost (i.e., it is efficient). As a result, it can be granted with the provision of those services for a subsequent period; even though it asks the public authority for a relatively high transfer to provide the public services. It may happen because the entrant firm, the outside option for the local government, does not benefit from learning-by-doing, thereby having higher cost than its opponent.

As it turns out, learning-by-doing may explain the lack of competition in those industries. However, it does not fully explain the set of evidence, in particular, the increasing subsidies and relatively high prices charged by private firms for certain public services. We shall demonstrate that private firms' ability to transferring learning among concessions added to learning-by-doing is a candidate for explaining the remaining evidence.

In order to explain how these two pieces jointly determine the industry competition, we build upon a key observation distinguishing private from local public firms: private firms may operate several concessions, whereas local public ones only provide the local public service in its home city. This gives them an important advantage over public firms: private firms can transfer learning from concessions where they have accumulated learning-by-doing (i.e., where they are incumbent), to concessions where they have not (i.e., where they are entrant). Due to this transferability of learning, a private firm can have low cost (efficient), even though it is an entrant. By contrast, a public firm becomes efficient only if it is an incumbent.

To develop these ideas, we analyze a two-period model of competition between a private and a local public firm for provision of a local public good. Firms are symmetric with respect to the cost structure and, when they are incumbent, they may have lower production cost (i.e., efficient) due to learning-by-doing. The unique difference between the private firm and the public one is that the former is likely to have lower production cost in a second period provision, whenever it is incumbent or not, as it may have access to external learning.

By backwards, the second-period competition is either between symmetric or asymmetric firms with respect their production cost. The symmetric competition occurs between two efficient firms, an incumbent public firm and an entrant private firm with external learning. Yet the asymmetric one happens either between an incumbent public firm and an inefficient entrant private one without external learning; or between an incumbent private firm and an entrant public firm.

In the symmetric case, the two efficient firms fiercely compete for the local public good provision and, therefore, they have the same chance of winning the competition. However, both firms expect to have relatively low transfers, i.e., low expected profits, from the local public authority because they face a intense competitor. Yet in the asymmetric case, the incumbent firm will be the most efficient, therefore it will be granted with the public good provision with a higher probability than the entrant, even though it asks for relatively high transfer to provide the public good. Such pattern is consistent with evidence documented by Szymanski (1996) in the refuse collection services in UK.[1]

In a first-period competition, when choosing the optimal strategy, both firms anticipate the dynamic effect of winning in the first period on the second-period competition. In particular, the private firm notices that winning the first-period competition, it will be the most efficient competitor at the second-period competition. Consequently, it will certainly enjoy high rents

[1]Szymanski (1996) shows where private contractors are already established, competitive tendering is likely to continue. Where DSOs (public firms) have retained the contract, compulsory competitive tendering has had a relatively small impact.

in the second-period provision of the public good. By contrast, the public firm never enjoys such rents in the second-period competition because it may face the private firm with external learning as an efficient competitor. As it turns out, the private firm has higher benefits than the public one for winning the first-period competition.

The equilibrium outcome of this repeated competition will be the following: the private firm will be more aggressive than the public one in the first-period competition, which implies that the former will have higher probability of winning the first competition than the latter. Consequently, the private firms is likely to be the unique efficient firm in the subsequent competition. In such contingency, it will easily win the competition with the public firm, enjoying high monetary transfers, i.e., rents, in the second period of public good provision. Yet in the case that the public firm wins the first-period competition, the public firm is unlikely be an unique efficient firm in the second period, since it may face an efficient competitor - a private firm with external learning. Therefore, it will end up receiving a low monetary transfer from the public authority to provide the good because firms face a intense competition afterwards.

The existence of such transferability of learning in concessions and learning-by-doing has two different effects on the second-period consumers welfare. On the one hand, transferability of learning makes an entrant private firm as competitive as an incumbent public one, leading to a fierce competition. In such contingency, the public authority ends up making low monetary transfer to the winner, either private or public, to perform the public services. Indeed, in this case, private firms' transferability of learning has the role of reducing the monetary transfer to the public service provider, thereby increasing the consumer welfare.

On the other hand, transferability of learning makes the private firm more efficient than public one: private firm is the only one which has access to outside learning. As a consequence, the private firm will be more likely to win a first competition with a public firm, thereby being incumbent in a future competition. As incumbent the private firm is more likely to win a future competition with the (entrant) public firm, even thought it bids for a relatively high monetary transfer to run the public services. Hence, differently from the case before, transferability of learning increases the market power of private firm, leading to high transfer to private incumbent firm and low consumers welfare.

Summing up these two opposite effects, we can analyze the net effect of private firm's transferability on second-period consumers welfare. We show that when the private firm has low ability on transferring learning from different concessions, then private firms' transferability of learning increases consumers welfare. However, when the private firm's ability on transfer knowledge from different concessions is relatively high, then transferability decreases consumers welfare. As a result, the second-period consumers' gains from changing to private ownership are high in industries where private firms have low ability on transferring knowledge among different concessions.

These two effects help to understand the evidence that subsidies to private firms are higher than public ones documented by Bontemps, Martimort, Roucolle and Thomas (2009): transferability of learning bounds the monetary transfer to public firms, whereas it increases the expected transfer to private firms.

Nevertheless, the effect of transferability on first-period consumers welfare is negative. The reason is the following: Transferability reduces both public and private firm's marginal benefit of winning, thereby reducing their incentives to bid low monetary transfers to provide public services.

Adding the first-period effect to the second-period one, we can demonstrate that the total consumers welfare is decreasing in the private firm's transferability. However, in an extension where we introduce competition between private firms, we can show that private firm's transferability is welfare enhancing if and only if number of private firms is sufficiently high.

Shaoul (1997), analyzing the privatized firms in Water and Sewerage Companies of England and Wales, finds that the prices charged by private firms in their second-period contract of water provision is substantially higher than the prices charged by private firms in their first-period contract. This empirical evidence confirms the findings of our model as it can be interpreted as private firms being less aggressive after they won the provision once. So, given that excessive private firm's aggressiveness in the first period, one question follows: Is there a mechanism that a local authority could design to benefit from this excessive private firm's aggressiveness in the first period and increase consumer welfare?

We can show that a biased procurement auction in the first period, in favor of local public firm (less efficient), helps to extract the rent of the private one (more efficient) and leads to higher consumer welfare. This policy recommendation extends McAfee and McMillan (1989), as they show that favoring the domestic firm (less efficient) reduces the foreign firm's rent (more efficient) in an environment where the difference of efficiency is assumed to be exogenous.

Related Literature. Our paper is related to the literature on repeated competition for public services, started with Laffont and Tirole (1988), where the incumbent has learning-by-doing. Laffont and Tirole show that when the incumbent invests in private learning-by-doing, i.e., other firms do not benefit from incumbent's investment, a biased auction which favors the entrant is the optimal mechanism. Indeed, they show that a regulator which favors the less efficient firm (entrant) can reduce the rent of the most efficient one (incumbent), and increase the social welfare. Our results are similar to Laffont and Tirole, since we can show that consumer welfare is higher when the entrant and the less efficient (public) firms are favored in a competition for public services.

Other papers in the literature study situations in which an incumbent firm may be more efficient than an entrant in local public good provision. For instance, Aubert, Bontemps and Salanié (2005, 2006), studying the lack of competition in the water and sanitation services in France, argue that an incumbent accumulates through time valuable information on the state of the network, which creates a winner's curse during symmetric auctions for renewing the concession. In Aubert, Bontemps and Salanié, private and public firms are completely symmetric and, while incumbent firms, have the same chance of winning the auctions and equally explore the incumbent advantage to extract more rents. Differently, in our model the existence of private firm's transferability of learning creates asymmetry between firms, making the private firm more efficient. However, it reduces consumes welfare in industries with low degree of competition.

Our paper has some connections with the literature on market design, in particular on how to optimally structure the competition between asymmetric firms (bidders) in order to achieve high consumer welfare (high auction revenue). Maskin and Riley (2000) show that an efficient mechanism may not maximize consumer welfare. In particular, they show that an asymmetric auction which favors the less efficient firm (bidder) produce higher welfare than the standard symmetric auction. McAfee and McMillan (1989) have a similar result analyzing the competition (procurement auction) between a domestic-inefficient versus a foreign-efficient firm for a government contract. Consistently with Maskin and Riley, they find the government minimizes its expected procurement cost by operating a price-preference policy, in which domestic firms are favored vis-à-vis foreign ones.[2] As it turns out, our policy recommendation is consistent with Maskin and Riley and McAfee and McMillan.

Our paper is also related to the literature on public versus private ownership of public services. Some papers have studied the potential economic reasons for private firms to be more efficient than public ones. Shleifer (1998), for instance, argues that government-owned firms are rarely the appropriate firm ownership because the owners of public firms are less able to write complete contracts with their managers due to diffuse ownership, making it difficult to tie the manager's incentives to the returns from their decisions. Yet Hart, Shleifer and Vishny (1994) argue that private firms have stronger incentive to engage in both quality improvement and cost reduction than public firms. However, due the incompleteness of contracts, private firms' incentives to engage in cost reduction typically are too strong because they ignore the adverse effect on non-contractible quality.[3]

[2]Branco (1994), followed by Vagstad (1995) and Naegelen and Mougeot (1998), extends McAfee and McMillan to the case which foreign firms profits do not enter in domestic welfare. Their effect intensifies the benefits of favoritism already presented in McAffe and McMillan (1989).

[3]Sappinton and Stiglitz (1987) do not argue the private firms are more efficient than public one. Differently, they show that the privatization can be useful as a commitment device: the government cannot commit itself

Our paper contributes to this literature providing a new reason for private firms to be more efficient than local public ones. In our model, a private firm competes in several concessions, and then it enjoys some learning-by-doing which can be used to reduce cost in many concessions at the same time. Yet a local public firm does not have such scope advantages because it serves only their own local market. Those economies of scope make the private firms more efficient than public ones. Another interpretation of our contribution is the following: Competitive pressure from other markets obliges private firms to be efficient, whereas the nonexistence of outside markets for local public firms turns them obsolete.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the Bayesian Perfect Equilibrium of the game. Section 4 makes a welfare analysis, and Section 5 provides a set of policy recommendations which may improve the consumers' welfare. Section 6 concludes. Proofs of the propositions that are not in the text can be found in the appendix.

2 The Model

We consider a city-economy consisting of consumers, a local public authority, a private firm and a public firm. All agents are risk-neutral and live for two periods. For simplicity, assume that time discount factor is equal to 1.

2.1 Consumers and Local Public Authority

There is a continuum of identical consumers in this city such that they derive the same utility from the local public good provision. For simplicity, we assume that the sum of all consumers utility for one unit of public good is u per period. The public good is an indivisible good and must be provided at most by one firm per period of time. This assumption fits to the case of local public services like garbage collection, street repairing, fire departments, local public transportation, and local public goods like potable water, where common carriage is not technologically feasible.

The local public authority, which is assumed to be a benevolent institution, is responsible for choosing the provider of the public good. In addition, he collects tax from consumers to pay for the public good provision. Due to the exclusive provision feature of the good, the local public authority has to choose only one firm to provide the good in the city. In principle, at the beginning of the first period, the local authority could make an once-for-all decision,

not to intervene in the control of the public enterprise, whereas such commitment becomes possible if assets have been sold to private entrepreneurs because, for instance, enacting a new regulation takes time and is costly.

selecting the firm which will provide the public good for the lifetime of the economy, i.e., two periods. However, we assume there is an institutional constraint which requires that, at the beginning of each period, the authority selects his public good provider only for the period ahead.[4]

As a standard procedure in selection of public good providers, e.g., Laffont and Tirole (1993), we assume that, at the beginning of each period, the local authority organizes a first-price procurement auction to assign the one-period-public-good provision to one of the firms in the economy.[5] In the first-price procurement auction, firms bid for the monetary transfer that they want to receive for the one period public good provision. The firm with the lowest bid will be granted with the public good provision for one period and will receive from the local authority a monetary transfer which corresponds to the value of his bid.[6]

In the end of each procurement auction, the local public authority taxes consumers in order to pay for the public good.[7] The local authority collects the same amount of taxes from each consumer such that the total tax revenue is equal to monetary transfer to the public good provider in the period.

Hence, consumers derive utility u in period t and pay a monetary transfer p_t to the provider of the public good. Therefore, the ex-ante welfare of consumers is:

$$W = u - E[p_1] + u - E[p_2]. \tag{1}$$

where $u - E[p_1]$ and $u - E[p_2]$ are, respectively, the first and the second-period ex-ante consumers welfare.

2.2 Firms

There are two firms in the economy: a private firm and a public firm denoted by the superscript F and G respectively. All are endowed with a technology which makes them potential providers of the public good.

[4]Laffont and Tirole (1993) present some political economy arguments which explain the existence of institutional constraints limiting the long-term contracts in public good/ service provision. Ellman (2006) provides a theory for the optimal length of contracts in concessions.

[5]In some industries, as local transportation in France, firms bid for the public subsidies to perform the provision of the services. The analysis which will develop in this paper also applies for the case of bidding for subsidies.

[6]We are restricting the analysis to fixed-price contracts. That is equivalent to assume that firms' production cost is not verifiable.

[7]Here, the local public authority is a passive agent, who just grants the provision the firm with the lowest bid in the auction and collects taxes from consumers. In Section 5, we discuss how the local authority could act as a strategically agent who maximizes consumer welfare.

Production' Costs. To produce one unit of public good, firms incur production cost, which varies over time and can be different from one firm to another. A firm i to produce the good at the period t will incur a cost c_t^i, with $i = \{F, G\}$.

At the beginning of each period, the production cost of each firm i, c_t^i, is drawn. Firms privately learn their own production cost for the correspondent period. In the two periods the maximal cost is denoted $\overline{c}$ and we assume that $\overline{c}$ is lower than marginal utility u such that the public good will be provided in every period.

The firms' production costs in the first period c_1^i, are independently drawn from an uniform distribution function on $[\underline{c}, \overline{c}]$, with $0 < \underline{c} < \overline{c}$. At the beginning of the second period, the second period cost c_2^i is drawn. The second period costs are still unknown by the firms in the first period. Despite this uncertainty, firms in the first-period are not completely uninformed about their second-period costs because there is a learning process along of the game which enables a firm to have information about its own and its opponent's production costs in the second period. In particular, we assume that firms gain proficiency through repetition of an activity, i.e., learning-by-doing. Hence, firms become more efficient overtime performing the public good provision.

Following Fudenberg and Tirole (1983), a firm which produces the public good in the first period will be granted with an expected reduction in its second-period production cost. Formally, we assume that an incumbent firm-i will have his second-period production cost c_2^i drawn from an uniform distribution function on $[0, \widetilde{c}]$, with $\underline{c} < \widetilde{c} < \overline{c}$. On the contrary, if firm-i does not produce the good in the first period, it will have his second-period production cost c_2^i drawn from the first-period uniform distribution $U[\underline{c}, \overline{c}]$. As it turns out, the learning-by-doing makes the incumbent firm, either public or private, relatively more efficient than the potential entrant in the second-period competition.[8]

For simplicity, we assume that $\widetilde{c} = \overline{c} - \underline{c}$. Then learning-by-doing reduces the expected second-period production cost, however it keeps constant the variance of the production cost in both periods.

Firms' Objective and Strategies. The objective of both firms, private and public, is to maximize profits.[9] Every period firms compete in a procurement auction for provision of one

[8] In this paper, learning-by-doing exogenously determines the second-period cost. Hence, the choice of auction (i.e., first-price auction or second-price auction) does not affect the incentive to invest in learning-by-doing. In a more general setting in which firms invest in learning, the auction format may affect learning-by-doing. See, for instance, Arozamena and Cantillon (2004).

[9] This assumption may not sound natural to describe the behavior of a public firm. However, it fits well to the case which the public firm is operated by a manager who is privately informed about the public firm's production cost. A local public authority, which considers a public firm or a private one as potential provider of public services, will ask the public firm's manager for how much it will cost for the public firm to provide the activities. Certainly, the manager enjoys some private rents if he does not truthfully reveal the public

unit of the public good. The winner of the auction, firm-i with $i = \{F, G\}$, receives at period t a monetary transfer s_t^i, which corresponds to the value of its bid, and incurs a production cost c_t^i to produce the good. Hence, the winner's payoff in the period t is $s_t^i - c_t^i$. Yet the loser gets zero.

We impose the following assumption on the firms' bidding strategy:

Assumption 1 *Firms have linear bidding strategies such that*

$$s_t^i = a_t^i + b_t^i c_t^i, \text{ with } i = \{F, G\} \text{ and } t=1,2.$$

Firms have linear bidding strategy, i.e., bids are linear function of production cost. Assumption 1, added to the assumption that production costs are uniformly distributed, are very convenient in order to find closed form solution for the equilibrium bids and payoffs, as demonstrated by Krishna (2002).

The probability that firm-i wins the auction at period t is equal to the probability that its bid s_t^i is lower than its competitor bid s_t^{-i}. Hence, firm-i has expected profit in period t equals to

$$\pi_t^i(c_t^i, s_t^i, s_t^{-i}) = (s_t^i - c_t^i)Prob(s_t^i < s_t^{-i}). \tag{2}$$

Strategic firms choose their bids according to their production cost in other to maximize profits. Consequently, a firm-i, when computing his expected profit from the procurement auction, takes it into account that its competitor's bid (strategy) is a function its production cost, i.e., $s_t^{-i}(c_t^{-i})$. Replacing it in the equation (2), the firm-i's instantaneous expected profit in a procurement auction at period t will be the following:

$$\pi_t^i(c_t^i, s_t^i, s_t^{-i}(c_t^{-i})) = (s_t^i - c_t^i)Prob(s_t^i < s_t^{-i}(c_t^{-i})). \tag{3}$$

Public versus Private Firms. So far we have not made any distinction between public and private firms. In our model, we assume that there is only one difference between them: the public firm produces the public good only in the city, whereas the private firm can produce the good in the city and also elsewhere. This public-firm-home-provision assumption can be justified by the absence of competition between local public firms for provision of local public goods and local services. Yet the private-firm-wide-provision assumption is justified by the existence of private firms competing and providing public goods and services at the same time in several markets, either home or foreign markets.

firm's cost to the local public authority. The manager's private rents in this alternative approach is equivalent to profits of the public firm in a model with the assumption that public firm maximizes profits. As it turns out, the alternative approach is equivalent to the simple one analyzed in this paper.

The private firm's ability on competing and serving several markets may give it many advantages over the other firms, which have already been studied in the literature.[10] In this paper, we present another advantage which is transferability of learning. We argue that providing the public good in many cities (i.e., concessions) at the same time allows the private firm to transfer technology from cities where it is incumbent and, therefore, it has gained learning-by-doing, to cities where it is not. Because the private firm can transfer learning inside the firm, the private firm may have reduction in expected cost, even though it is entrant. By contrast, public firm only have cost reduction if it is incumbent.

In order to model transferability of learning, we assume that there exist two possible states of the world in the end of the first period. With probability θ, the private firm is the provider of the public good (incumbent) elsewhere, therefore it expects to have lower production cost in the second period: c_2^F is distributed according to an uniform on $[0, \widetilde{c}]$. With probability $1-\theta$, it is not good provider elsewhere. Hence, it expects to have the second period cost c_2^F distributed according to an uniform on $[\underline{c}, \overline{c}]$, as in the first period.

Note that, the higher the probability θ, the higher the probability that the private firm transfers learning inside of the firm. The probability θ is then a measure of the private firm's ability on transferring learning between different cities.

However, the advantage of the private firm over the public one, θ, somehow also measures the degree of competitiveness in the economy: the higher θ, the higher the probability that an incumbent public firm will face an efficient private firm in the second period, which leads to a fierce second-period competition. The equilibrium conditions will determine which values of θ are pro and counter-competitive.

Firms' Payoffs. Due to the dynamics of the game, we will describe the firms' payoffs by backwards. So, we first describe the second-period payoffs, and then we present the first-period ones.

In the second period, there are four possible contingencies when firms compete in procurement auction. To reduce notation, all possible contingencies are summarized by a state variable X with three possible states.

- **State $X = 1$: Incumbent Public Firm and Private Firm without Transferability of Learning.** The public firm was public good provider in the city and the private firm was not incumbent elsewhere in the first period. The public firm expects to have lower production cost in the second period, whereas the private firm does not because it has not access to external learning. Hence, public firm's second-period cost c_2^G will be

[10] Tirole (1988) discusses the economies of scope and scale in multi-product and multi-market firms.

drawn from the uniform distribution on $[0, \tilde{c}]$, and the private firm's second-period cost c_2^F will be drawn from the uniform distribution on $[\underline{c}, \overline{c}]$.

- **State $X = 2$: Incumbent Public Firm and Private Firm with Transferability of Learning.** The public firm was public good supplier in the city and the private firm was incumbent elsewhere. Both firms expects to have lower second-period cost. Therefore, firms' second-period cost, c_2^F and c_2^G, will be drawn from the uniform distribution on $[0, \tilde{c}]$.

- **State $X = 3$: Incumbent Private Firm.** The private firm was the public good provider in the city and also elsewhere; or the private firm was the public good provider in the city, but not elsewhere. In both cases, the private firm's second-period cost c_2^F will be drawn from the uniform distribution on $[0, \tilde{c}]$, and public firm's second-period cost c_2^G will be drawn from the uniform distribution on $[\underline{c}, \overline{c}]$. These two contingencies are equivalent with respect to the distribution of firms' second-period costs.

Since firm-i's second-period payoff depends on the State X, we can write firm-i payoff in the second period, described generically in (3), as the function $\pi_2^i(.)$ characterized the following equation:

$$\pi_2^i(c_2^i, s_2^i, s_2^{-i}(c_2^{-i}), X) = (s_2^i - c_2^i) Prob(s_2^i < s_2^{-i}(c_2^{-i})|X). \tag{4}$$

In state X, firm-i chooses a bid s_2^i that maximizes its second-period expected payoff, described in (4). Hence, doing so firm-i earns the following expected second-period payoff:

$$\Pi_2^i(c_2^i, s_2^{-i}(c_2^{-i}), X) = \max_{s_2^i} \pi_2^i(c_2^i, s_2^i, s_2^{-i}(c_2^{-i}), X). \tag{5}$$

Now, let us turn to the description of firms' first-period payoff and decision. In the first period, at the moment that firms compete in procurement auction and decide their bids, they only know their own production cost and that its opponent's production cost is drawn from an uniform distribution on $[\underline{c}, \overline{c}]$. Hence, firm-$i$ with first-period production cost equal to c_1^i, expects instantaneous profit at period 1 equals to $\pi_1^i(c_1^i, s_1^i, s_1^{-i}(c_1^{-i}))$, defined in (3).

Firm-i chooses s_1^i that maximizes the first-period profit plus the expected second-period (i.e., continuation payoff). This problem is expressed in the following equation:

$$\Pi^i(c_1^i, s_1^{-i}(c_1^{-i})) = \max_{s_1^i} \left\{ \pi_1^i(c_1^i, s_1^i, s_1^{-i}(c_1^{-i})) + E\left[\Pi_2^i(c_2^i, s_2^{-i}(c_2^{-i}), X)|s_1^i, s_1^{-i}(c_1^{-i})\right] \right\} \tag{6}$$

where $E\left[\Pi_2^i(c_2^i, s_2^{-i}(c_2^{-i}), X)|s_1^i, s_1^{-i}(c_1^{-i})\right]$ is the firm-i's second-period expected, given its bidding strategy s_1^i and its opponent bidding strategy $s_1^{-i}(c_1^{-i})$ in the first period.

Note that, in equation (6) the first-period bidding strategy s_1^i affects the second-period payoff through X, thereby determining the second-period state. Due to this intertemporal effect of first-period bid, firms face a dynamic trade-off between profits in the first and profits in the second period: Bidding low in the first period reduces its first-period profit, however it increases the probability of winning in the first period. With high probability of winning in the first period, it will be likely to have low second-period cost and, therefore, high second-period profit.

2.3 Timing

First Period

(i) The nature draws firms' production cost for the first period. All costs are drawn from an uniform on $[\underline{c}, \overline{c}]$.

(ii) Each firm privately learns its own first-period cost. Firms send their bids to the local public authority for one period provision of public good.

(iii) The firm with the lowest bid wins the auction, provides the public good and receives a monetary transfer, which corresponds to the value of its bid.

Second Period

(iv) The nature draws private firm's transferability: (a) with probability θ, the private firm will be able to transfer efficient technology from elsewhere to this city, independently of the first-period outcome; (b) with probability $1 - \theta$, the private firm will not be able to transfer efficient technology from elsewhere, its second-period cost will depend on the first-period outcome.

(v) The nature draws firms' production cost for the second period. In the case that a firm is incumbent in the city, or it is a private firm with transferability of learning, the firm's production cost will be drawn from an uniform on $[0, \widetilde{c}]$. Otherwise, the production cost will be drawn form uniform on $[\underline{c}, \overline{c}]$.

(vi) Each firm privately learns its own second-period cost. Firms send their bids to the local public authority for one period provision of public good.

(vii) The firm with the lowest bid wins the auction and produce the public good in the city.

3 The Equilibrium Analysis

The model is a dynamic game with asymmetric information since firms are privately informed about their production cost and learn some information about their own and opponent future cost over time. In this section, we will look for the Perfect Bayesian Equilibrium (PBE) of the game.

In order to characterize the equilibrium we will solve the model by backward induction: (i) we find the Bayesian Nash Equilibrium (BNE) in each possible contingency of the second-period competition, and (ii) we turn to the characterization of the BNE in first-period competition.

3.1 Second-Period Competition

In this subsection we characterize the BNE in each contingency of the second period, and the correspondent firms' expected profit in equilibrium. The second-period competition takes place under three possible contingencies, which were described in the previous section. The remainder of this subsection characterizes the equilibrium in each of those contingencies, and describes firms' payoff.

■ **State $X = 1$.** At the proceeding state, at date v, firms are not symmetric in this competition. For this reason, we first analyze the public firm's behavior, and then we analyze the private's one.

According to (4), the public firm has expected profit equal to

$$\pi_2^G = (s_2^G - c_2^G) Prob(s_2^G < s_2^F(c_2^F)|X = 1).$$

Under Assumption 1, the public firm's expected profit is such that:

$$\pi_2^G = (s_2^G - c_2^G) Prob(s_2^G < a_2^F + b_2^F c_2^F|X = 1). \tag{7}$$

In the contingency analyzed in this subsection, c_2^F is distributed according to uniform distribution on $[\underline{c}, \overline{c}]$. After replacing the cumulative distribution in (7) and some algebraic manipulations, we obtain the following public firm's expected profit:

$$\pi_2^G = \frac{s_2^G - c_2^G}{\overline{c} - \underline{c}} \left[\overline{c} - \frac{s_2^G - a_2^F}{b_2^F} \right]. \tag{8}$$

The public firm chooses its bidding strategy s_2^G that maximizes (8). The first-order conditions of this maximization problem delivers the following public firm's optimal bidding strategy:

$$s_2^G = \frac{b_2^F \overline{c} + a_2^F}{2} + \frac{1}{2} c_2^G.$$

Having derived the public firm's payoff and strategy, let us turn to the characterization of the private firm's ones. Under Assumption 1, the private firm's expected profit is given by:

$$\pi_2^F = (s_2^F - c_2^F) Prob(s_2^F < a_2^G + b_2^G c_2^G | X = 1). \tag{9}$$

In the contingency analyzed in this subsection, c_2^G is distributed according to uniform distribution on $[0, \widetilde{c}]$, i.e., the public firm expects to have lower cost in the second period. Replacing the cumulative distribution in (9) and making some algebraic manipulations, we obtain the following private firm's expected profit:

$$\pi_2^F = \frac{s_2^F - c_2^F}{\widetilde{c}} \Big[\widetilde{c} - \frac{s_2^F - a_2^G}{b_2^G} \Big]. \tag{10}$$

The private firm chooses its bidding strategy s_2^G that maximizes (10). The first-order conditions of this maximization problem delivers the following private firm's optimal bidding strategy:

$$s_2^F = \frac{b_2^G \widetilde{c} + a_2^G}{2} + \frac{1}{2} c_2^F.$$

Finally, we characterize the Bayesian Nash equilibrium of this stage game. The Bayesian Nash Equilibrium is the profile of bidding strategy functions $\big(s_2^{*F}(c_2^F), s_2^{*G}(c_2^G)\big)$, such that, $s_2^{*i}(c_2^i)$ is the best response for $s_2^{*-i}(c_2^{-i})$, with $i = F, G$.

Under Assumption 1, the private and public firm's equilibrium bidding strategies are characterized, respectively, by the following functions:

$$s_2^{*F}(c_2^F, X = 1) = \frac{2\widetilde{c} + \overline{c}}{6} + \frac{1}{2} c_2^F, \tag{11}$$

$$s_2^{*G}(c_2^G, X = 1) = \frac{2\overline{c} + \widetilde{c}}{6} + \frac{1}{2} c_2^G. \tag{12}$$

In equilibrium the parameters a_2^i and b_2^i of the bidding functions described by (11) and (12) take the following values: $b_2^i = \frac{1}{2} \forall i$, $a_2^F = \frac{2\widetilde{c}+\overline{c}}{6}$ and $a_2^G = \frac{2\overline{c}+\widetilde{c}}{6}$. Replacing them in equations (8) and (10), we obtain the equilibrium expected profits:

$$\pi_2^{*F}(c_2^F, X = 1) = \frac{1}{2\widetilde{c}} \Big[\frac{2\widetilde{c} + \overline{c}}{3} - c_2^F \Big]^2, \tag{13}$$

$$\pi_2^{*G}(c_2^G, X = 1) = \frac{1}{2(\overline{c} - \underline{c})} \Big[\frac{2\overline{c} + \widetilde{c}}{3} - c_2^G \Big]^2. \tag{14}$$

The equilibrium in State $X = 1$ described by equations (11) and (12) has the following properties:

Lemma 1 *Let $\tau_2^i(X)$ be the probability of firm-i wins the auction in the state X. In State $X = 1$,*

(i) public firm's bidding function is strictly greater than private firm's bidding function: $s_2^{*G}(.) > s_2^{*F}(.)$;

(ii) the public firm is more likely to win the auction than the private one, $\tau_2^G(X = 1) > \tau_2^F(X = 1)$, such that $\tau_2^G(X = 1) = \frac{18\widetilde{c}(\widetilde{c}-\underline{c})-(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$ and $\tau_2^F(X = 1) = \frac{(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$.

Property (i) in Lemma 1 says that in the case that the public and private firms have the same production cost, the public firm's bid is higher than the private one. It means that the incumbent public firm is less aggressive than its competitor, the entrant private firm. Intuitively, the public firm knows that is likely to have lower cost than its competitor. Hence, it does not need to be too aggressive in the competition in order to win it.

Despite the fact that the public firm is not very aggressive in the competition (i.e., higher bidding function), it has higher probability of winning the auction than the private one. That is Property (ii) in Lemma 1. In summary, the incumbent firm has the highest probability of winning, even though it asks the public authority a higher monetary transfer to provide the public good.

■ **State $X = 2$.** In the second state, firms are symmetric. Hence, we analyze the payoff and behavior of a generic firm-i, with i being equal to F for private firm, and G for public firm. Under Assumption 1 and given that c_2^{-i} is distributed according to the uniform distribution on $[0, \widetilde{c}]$ for all i, firm-i's expected profit is characterized by:

$$\pi_2^i = \frac{s_2^i - c_2^i}{\widetilde{c}} \left[\widetilde{c} - \frac{s_2^i - a_2^{-i}}{b_2^{-i}} \right], \text{ for all } i. \tag{15}$$

Firm-i chooses its bidding strategy s_2^i that maximizes (15). The first-order conditions of this maximization problem delivers the following firm-i optimal bidding strategy:

$$s_2^i = \frac{b_2^{-i}\widetilde{c} + a_2^{-i}}{2} + \frac{1}{2}c_2^i.$$

The equilibrium bidding strategies are characterized by the following functions:

$$s_2^{*i}(c_2^i, X = 2) = \frac{\widetilde{c}}{2} + \frac{1}{2}c_2^i, \text{ for all } i. \tag{16}$$

In equilibrium the parameters a_2^i and b_2^i of the bidding functions, described in (16), take the following values: $b_2^i = \frac{1}{2}$ and $a_2^i = \frac{\widetilde{c}}{2}$ for all i. Replacing them in equation (16), we obtain the equilibrium expected profit:

$$\pi_2^{*i}(c_2^i, X=2) = \frac{(\widetilde{c} - c_2^i)^2}{2\widetilde{c}}, \text{ for } i = F, G. \tag{17}$$

Equation (16) describes the equilibrium in State $X = 2$. The equilibrium properties are described in the following lemma:

Lemma 2 *In State $X = 2$, firms' bidding functions and their probabilities of winning are the same, such that $s_2^{*G}(.) = s_2^{*F}(.)$ and $\tau_2^G(X=2) = \tau_2^F(X=2)$.*

Lemma 2 says that if both firms have the same production cost, then they have the same bid and the same probability of winning the auction. It is quite intuitive since in this state, firms are perfectly symmetric.

■ **State $X = 3$.** The procurement auction game between the public and the private firm in this state is equivalent to the one analyzed in State $X = 1$. The only difference is that private firm's expected payoff when $X = 3$ coincides with public firm's expected payoff when $X = 1$, and vice-versa.

Due to this symmetry, equilibrium bidding strategy of the private firm when $X = 3$ will be described by (12), and the public firm's one will be described by (11). Hence, the private and public firm's equilibrium bidding strategies are, respectively, characterized by:

$$s_2^{*F}(c_2^F, X=3) = \frac{2\overline{c} + \widetilde{c}}{6} + \frac{1}{2}c_2^F, \tag{18}$$

$$s_2^{*G}(c_2^G, X=3) = \frac{2\widetilde{c} + \overline{c}}{6} + \frac{1}{2}c_2^G. \tag{19}$$

The equilibrium expected profits when $X = 3$ are characterized by the following expressions:

$$\pi_2^{*F}(c_2^F, X=3) = \frac{1}{2(\overline{c} - \underline{c})}\left[\frac{2\overline{c} + \widetilde{c}}{3} - c_2^F\right]^2, \tag{20}$$

$$\pi_2^{*G}(c_2^G, X=3) = \frac{1}{2\widetilde{c}}\left[\frac{2\widetilde{c} + \overline{c}}{3} - c_2^G\right]^2. \tag{21}$$

The equilibrium properties of the BNE described by equations (18) and (19) are described in the following lemma:

Lemma 3 *In State $X = 3$,*

(i) private firm's bidding function is strictly greater than public firm's bidding function $s_2^{*F}(.) > s_2^{*G}(.)$*;*

(ii) the private firm is more likely to win the auction than the public one, $\tau_2^F(X=3) > \tau_2^G(X=3)$*, such that* $\tau_2^F(X=3) = \frac{18\widetilde{c}(\overline{c}-\underline{c})-(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$ *and* $\tau_2^G(X=3) = \frac{(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$.

Property (i) in Lemma 3 says that incumbent private firm is less aggressive than its competitor, the entrant public firm. The explanation for this behavior is the same as described in Lemma 1: the private firm knows that is likely to have lower cost than its competitor, therefore, it does not need to be too aggressive to win the competition. Yet Property (ii) in Lemma 3 says that even tough the private firm is not very aggressive in the competition, i.e., higher bidding function when the cost are equal, it has higher probability of winning the auction.

As it turns out, Lemmas (1) to (3) provides the following testable implication:

Implication 1 *In the second-period competition, the incumbent firm has higher probability of winning than the entrant.*

Implication 1 arises due to the existence of learning-by-doing. It can be interpreted as follows: the higher firm's learning-by-doing in a certain industry, the more likely that the incumbent firm wins the competition with the entrant.[11] It is consistent with GEA-ENGREF (2002), a recent report on contracts of water concession in France, which documents that in 78 % of auctions for concession, the incumbent is never replaced. The water sector is recognized as a sector which the incumbent enjoys learning-by-doing, as described by Aubert, Bontemps and Salanié (2005, 2006). It is also consistent with the evidence documented by Szymanski (1996) in the refuse collection services in UK. Szymanski shows that where private contractors are already established, competitive tendering is likely to continue. Where DSOs (public firms) have retained the contract, compulsory competitive tendering has had a relatively small impact.

3.2 First-Period Competition

Having characterized the BNE in all possible second-period contingencies, we should turn to the characterization of the first-period competition. However, as described in equation (6), in order to characterize firms' first-period bidding strategy, we need to characterize firms'

[11] It may be complicated to measure the degree of learning-by-doing in a certain industry, which makes the test of our theory quite difficult. However, learning-by-doing can be related to the duration that a firm is providing a certain public service. In this case, the prediction of Implication 1 can be rewritten as follows: the longer the period the incumbent is the provider of the public good, the higher is the probability that it will win the competition with the entrant firm for a new contract of public service provision.

continuation payoff after the first period, i.e., firms' second-period expected equilibrium payoff. The following Lemma summarizes firms' first-period continuation payoff.

Lemma 4 *Let $\Pi_2^i()$ be the first-period continuation payoff function of firm-i. The first-period continuation payoff of the private and public firms are such that*

(i) if the private firm wins the first-period auction, then

$$\Pi_2^F(s_1^F < s_1^G) = \overline{\Pi},$$

$$\Pi_2^G(s_1^F < s_1^G) = \underline{\Pi},$$

such that $\overline{\Pi} := \frac{4\bar{c}^2+\tilde{c}^2-2\bar{c}^2}{18\tilde{c}}$ *and* $\underline{\Pi} := \frac{4\tilde{c}^2+\bar{c}^2-2\bar{c}\tilde{c}+3\underline{c}^2-6\bar{c}^2}{18\tilde{c}}$*;*

(ii) if the public firms wins the first-period auction, then

$$\Pi_2^F(s_1^F > s_1^G) = \theta\Pi^C + (1-\theta)\underline{\Pi},$$

$$\Pi_2^G(s_1^F > s_1^G) = \theta\Pi^C + (1-\theta)\overline{\Pi},$$

such that $\Pi^C := \frac{\tilde{c}}{6}$.

In the first period of game, the public and private firm bid for the first-period provision of the public good. In order to derive firms' first-period strategy in this procurement auction, let us analyze each firm decision separately. We first describe private firm's behavior and then we turn to the public firm's one.

In the case that the private firm wins the first-period competition, which happens if $s_1^F < s_1^G$, it receives a monetary transfer equal to s_1^F and incurs a production cost c_1^F. Hence, it will have an instantaneous payoff equal to $s_1^F - c_1^F$. In addition to that, the private firm will have a continuation payoff equal to $\overline{\Pi}$, as described in Lemma 4. Therefore, if the private firm wins the first-period competition it obtains a payoff equal to $s_1^F - c_1^F + \overline{\Pi}$. If the private loses the first-period competition, it has zero payoff in the first period. Nevertheless, according to Lemma 4, it has continuation payoff equals to $\theta\Pi^C + (1-\theta)\underline{\Pi}$. Hence, the private firm's expected payoff in the first-period competition is the following:

$$\Pi_1^F = (s_1^F - c_1^F + \overline{\Pi})Prob(s_1^F < s_1^G) + (\theta\Pi^C + (1-\theta)\underline{\Pi})Prob(s_1^F \geq s_1^G). \tag{22}$$

Under Assumption 1, private firm's first-period expected profit is as follows:

$$\Pi_1^F = (s_1^F - c_1^F + \overline{\Pi})Prob(s_1^F < a_1^G + b_1^G c_1^G) + (\theta\Pi^C + (1-\theta)\underline{\Pi})Prob(s_1^F \geq a_1^G + b_1^G c_1^G). \tag{23}$$

As c_1^G is distributed according to uniform distribution on $[\underline{c}, \overline{c}]$, private firm's expected profit Π_1^F is given by

$$\Pi_1^F = \frac{s_1^F - c_1^F + \overline{\Pi}}{\overline{c} - \underline{c}} \Big[\overline{c} - \frac{s_1^F - a_1^G}{b_1^G}\Big] + \frac{\theta \Pi^C + (1-\theta)\underline{\Pi}}{\overline{c} - \underline{c}} \Big[\frac{s_1^F - a_1^G}{b_1^G} - \underline{c}\Big]. \tag{24}$$

The private firm chooses its bidding strategy s_1^F that maximizes (24). The first-order conditions of this maximization problem delivers the following private firm's optimal first-period bidding strategy:

$$s_1^F = \frac{a_1^G + b_1^G \overline{c}}{2} + \frac{\theta \Pi^C + (1-\theta)\underline{\Pi} - \overline{\Pi}}{2} + \frac{1}{2} c_1^F \tag{25}$$

Now, let us turn to the analysis of public firm's behavior. In the case that the public firm wins the first-period competition, which happens if $s_1^F > s_1^G$, it gets instantaneous payoff equal to $s_1^G - c_1^G$, where s_1^G is the monetary transfer that it receives to provide the public good and c_1^G is its production cost. In addition to that, the public firm will have a continuation payoff equal to $\theta \Pi^C + (1-\theta)\overline{\Pi}$, as described in Lemma 4. Therefore, if the public firm wins the first-period competition it obtains a payoff equal to $s_1^G - c_1^G + \theta \Pi^C + (1-\theta)\overline{\Pi}$. If the public firm loses the first-period competition, then it has zero payoff in the first period. Nevertheless, according to Lemma 4, it has continuation payoff equal to $\underline{\Pi}$. Hence, the public firm's expected payoff in the first-period competition is the following:

$$\Pi_1^G = (s_1^G - c_1^G + \theta \Pi^C + (1-\theta)\overline{\Pi}) Prob(s_1^F > s_1^G) + \underline{\Pi} Prob(s_1^F \leq s_1^G). \tag{26}$$

Following the same steps that we did in order to derive private firm's expected payoff, we obtain the following public firm's expected first-period profit:

$$\Pi_1^G = \frac{s_1^G - c_1^G + \theta \Pi^C + (1-\theta)\overline{\Pi}}{\overline{c} - \underline{c}} \Big[\overline{c} - \frac{s_1^G - a_1^F}{b_1^F}\Big] + \frac{\underline{\Pi}}{\overline{c} - \underline{c}} \Big[\frac{s_1^G - a_1^F}{b_1^F} - \underline{c}\Big]. \tag{27}$$

The public firm chooses its bidding strategy s_1^G that maximizes (27). The first-order conditions of this maximization problem delivers the following public firm's optimal first-period bidding strategy:

$$s_1^G = \frac{a_1^F + b_1^F \overline{c}}{2} + \frac{\underline{\Pi} - \theta \Pi^C - (1-\theta)\overline{\Pi}}{2} + \frac{1}{2} c_1^G \tag{28}$$

Once we described firms' bidding strategy, let us characterize the BNE of first-period auction. The private and public equilibrium bidding strategies are, respectively, characterized by the following functions:

$$s_1^{*F} = \frac{\overline{c}}{2} + \frac{\theta\Pi^C + \underline{\Pi}(3-2\theta) - \overline{\Pi}(3-\theta)}{2} + \frac{1}{2}c_1^F \tag{29}$$

$$s_1^{*G} = \frac{\overline{c}}{2} + \frac{-\theta\Pi^C + \underline{\Pi}(3-\theta) - \overline{\Pi}(3-2\theta)}{2} + \frac{1}{2}c_1^G \tag{30}$$

Equations (29) and (30) describe the equilibrium of the first-period. Replacing the equilibrium strategies above in the profit functions in (24) and in (27), we obtain the equilibrium private's firm expected profit Π_1^{*F} and public's firm expected profit Π_1^{*G} in the game.

The equilibrium has some properties which are described in the following proposition:

Proposition 1 *In the first-period procurement auction,*

(I) private firm's bidding function is strictly lower than public firm's bidding function: $s_1^{*F}(.) < s_1^{*G}(.)$*;*

(II) the private firm is more likely to win the auction such that $\tau_1^F > \tau_1^G$*. In particular,*

(i) If $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} < \frac{8}{27}$*, then there exists* $\widehat{\theta} \in (0,1)$ *such that for:*

(a) $\theta \geq \widehat{\theta}$, $\tau_1^F = 1$*;*

(b) $\theta < \widehat{\theta}$, $\tau_1^F = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$.

(ii) If $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} \geq \frac{8}{27}$*, then* $\tau_1^F = \frac{1}{2(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta(\overline{\Pi}+\underline{\Pi}-2\Pi^C)}{3}\right]^2$.

Property (I) in Proposition 1 says that if both firms have the same first-period production cost, the private firm's bid is lower than public's one. It means that the private firm is more aggressive than the public in the first-period competition. The intuition for this result is the following: The private firm notices that winning the first-period competition, it will be the most efficient competitor at the second-period competition. Consequently, it will enjoy high rents in the second period of provision of the public good. By contrast, the public firm never enjoys such rents in the second-period competition because it is likely to face the private firm with external learning. Then as the private firm has higher benefits than the public one for winning the first-period competition, the private firm will have lower bid than the public firm in the first-period auction.

If that the private firm's bid is lower than the public firm and firms are symmetric with respect to the production cost, then private firm's probability of winning will be higher than the public's one. That is exactly the result of Property (II) in Proposition 1. In particular, Property (II.i) in Proposition 1 shows if the private firm's transferability is sufficiently high ($\theta \geq \widehat{\theta}$) and the variance of first-period production cost is relatively low ($\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} < \frac{8}{27}$), then private firm will be so aggressive that it will win the first-period competition with probability one.

Of course, this asymmetry between private and public firms comes from the ability of private firms on transferring learning from different markets, which is measure by the parameter θ in the model. Therefore, we analyze the effect of θ on firm's equilibrium bidding strategies and on each firm's probability of winning. That is developed in the following proposition:

Proposition 2 *The effect of private firm's transferability θ on firm's equilibrium bidding strategies and firm's probability of winning are the following:*

(i) Public firm's first-period bid $s_1^{*G}(.)$ increases with private firm's transferability: $\frac{\partial s_1^{*G}(.)}{\partial \theta} > 0$.

(ii) If $\overline{c} > \frac{7\underline{c}}{5}$, then private firm's first-period bid $s_1^{*F}(.)$ increases with private firm's transferability, $\frac{\partial s_1^{*F}(.)}{\partial \theta} > 0$. Otherwise, private firm's first-period bid $s_1^{*F}(.)$ decreases with θ, $\frac{\partial s_1^{*F}(.)}{\partial \theta} < 0$.

(iii) The effect of Private firm's transferability θ is higher on public firm's bid than on private firm's bid such that $\frac{\partial s_1^{*G}(.)}{\partial \theta} > \frac{\partial s_1^{*F}(.)}{\partial \theta}$.

(iv) Private's (respectively Public) firm probability of winning in the first-period auction increases (respectively decreases) with θ such that $\frac{\partial \tau_1^F}{\partial \theta} \geq 0$ and $\frac{\partial \tau_1^G}{\partial \theta} \leq 0$.

Proposition 2 shows that the private firm's advantage over the public one increases with private firm's transferability. Property (i) says that transferability increases public and private firm's bid, i.e., it makes firms less aggressive in the first-period competition. The reason is the following: private firm's transferability reduces the public firm's winning continuation payoff because it will more likely that it will face the private firm with outside learning in the second-period competitor. Since private firm's transferability reduces the public firm's marginal benefit of winning the first-period, then public firm will bid less aggressive, thereby increasing its bid.

Naturally, the private firm anticipates that its competitor will be less aggressive in equilibrium. For this reason, in the case that variance of first-period production cost is relatively low ($\overline{c} < \frac{7\underline{c}}{5}$), the private firm will bid more aggressive in order to win the competition with probability equal or close to one. That is what Property (ii) in Proposition 2 shows.

Property (iii) in Proposition 2 says that transferability increases more the public firm's bid than the private firm's one. That happens because the public firm's expected payoff reduces when it faces an efficient private firm (high θ), what makes it to increase its bid. The private firm's expected payoff increase when θ increases, what, on the one hand, induces it to reduce its bid. However, private firm notices that in equilibrium the public firm increases its bid. Therefore, it realizes that to win he does not need to reduce its bid too much, thereby increasing its bid a little bit.

Property (iv) says that private's firm probability of winning increases with transferability θ, whereas public's firm probability of winning decreases with transferability θ.

4 Welfare Analysis

In order to understand the effect of private firm's transferability θ on ex-ante consumers welfare, defined in (1), we first study the effect of transferability on first-period expected transfer, and then we turn to the effect on the second-period's one. We will then determine whether private firm's transferability θ is pro-competitive (lowers monetary transfer to the public good provider) or counter competitive (increases the monetary transfer to the private firm).

4.1 First and Second Period Expected Transfers

First-period Expected Transfer. The first-period expected transfer $E[p_1]$, the transfer that the local public authority expects to pay to the winner of the first-period auction, is given by the following expression:

$$E[p_1] = \tau_1^G E[s_1^{*G}] + \tau_1^F E[s_1^{*F}], \tag{31}$$

where τ_1^G, defined in Proposition 1, is the probability that the public firm wins the first-period auction. Hence, the local public authority expects to pay to public firm its bid s_1^{*G}, defined in (30). Yet with probability τ_1^F, also defined in Proposition 1, the private firm wins the first-period auction. In that case, the local public authority expects to pay to the private firm its bid s_1^{*F}, defined in (29).

The following Lemma shows that private firm's transferability θ increases first-period expected transfer and, consequently, reduces first-period consumers welfare.

Lemma 5 *If $\frac{(\bar{c}-\underline{c})^2}{\underline{c}^2} \geq \frac{4}{9}$, then the first-period expected transfer $E[p_1]$, defined in (31), increases with private firm's transferability: $\frac{\partial E[p_1]}{\partial \theta} > 0$.*

Lemma 5 follows the results of Proposition 2. If the variance of first-period production cost is relatively high, then both firms' bid increases with θ. Since all bids increase with θ, it is natural that the expected tariff paid for the public good provider also increases with θ.

Second-Period Expected Transfer. The ex-ante second-period expected transfer $E[p_2]$ is the transfer that the local public authority expects to pay to the winner of the second-period auction. The computation of that expectation is more complicated than the first-period one because there are different possible states in the second-period, characterized by the state variable X in Section 3.1. The details are relegated to the Appendix.

Hence, the ex-ante second-period expected transfer is the following:

$$E[p_2] = \tau_1^F \overline{p} + \tau_1^G [(1-\theta)\overline{p} + \theta \underline{p}], \tag{32}$$

where $\overline{p} := \frac{4\overline{c}+5\widetilde{c}}{12}\left[\frac{18\widetilde{c}(\overline{c}-\underline{c})-(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}\right] + \frac{4\widetilde{c}+5\overline{c}+3\underline{c}}{12}\left[\frac{(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}\right]$ and $\underline{p} := \frac{3\widetilde{c}}{4}$.

The following proposition analyzes the effect of private firm's transferability θ on the ex-ante second-period expected transfer.

Proposition 3 *The second-period expected transfer $E[p_2]$ is such that:*

(I) When $0 \leq \frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} < \frac{8}{27}$, there exist $\widetilde{\theta}$ and $\widehat{\theta}$ such that

- *(i) for all $0 \leq \theta < \widetilde{\theta}$, $E[p_2]$ decreases with private firm's transferability: $\frac{\partial E[p_2]}{\partial \theta} < 0$;*
- *(ii) for all $\widetilde{\theta} \leq \theta < \widehat{\theta}$, $E[p_2]$ increases with private firm's transferability: $\frac{\partial E[p_2]}{\partial \theta} > 0$;*
- *(iii) for all $\theta \geq \widehat{\theta}$, $E[p_2]$ does not depend on private firm's transferability: $\frac{\partial E[p_2]}{\partial \theta} = 0$.*

(II) When $\frac{8}{27} \leq \frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} < \frac{8}{9}$, then there exists $\overline{\theta}$ such that

- *(iv) for all $0 \leq \theta < \overline{\theta}$, $E[p_2]$ decreases with private firm's transferability: $\frac{\partial E[p_2]}{\partial \theta} < 0$;*
- *(v) for all $\theta \geq \overline{\theta}$, $E[p_2]$ increases with private firm's transferability: $\frac{\partial E[p_2]}{\partial \theta} > 0$.*

(III) When $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} \geq \frac{8}{9}$, $E[p_2]$ decreases with private firm's transferability: $\frac{\partial E[p_2]}{\partial \theta} < 0$.

Proposition 3 shows that the existence of such transferability within concessions has two opposite effects on second-period expected transfer. On the one hand, transferability turns an entrant private firm as efficient (competitive) as an incumbent public one, leading to a fierce competition between these firms. As result, the public authority ends up making low transfer to the winner, either private or public, to perform the public services. Indeed, in this case, transferability has the role of reducing the monetary transfer to a second-period public good provider, thereby increasing the second-period expected consumer welfare.

On the other hand, transferability makes the private firm more efficient than public one: the private firm has access to outside learning, whereas the other one does not. As a consequence, the private firm will be more likely to win a first competition with a public firm, thereby being incumbent in a future competition. As an incumbent, the private firm is more likely to win a second competition with the (entrant) public firm, even thought its bids a relatively low transfer to run the public good provision. Hence, differently from the case before, transferability increases the market power of private firm, leading to high transfer and low second-period expected consumers welfare.

Summing up these two opposite effects, we have the net effect of private firm's transferability on consumer welfare. When the private firm has low ability on transfer knowledge from different markets (low θ), then private firms' transferability of learning increases ex-ante second-period expected consumers welfare. However, when the private firm's ability on transfer knowledge from different markets is relatively high (high θ), then transferability's effect decreases ex-ante second-period expected consumers welfare.

As it turns out, we can conclude that when θ is lower than the threshold ($\widehat{\theta}$ in the case that $0 \leq \frac{(\bar{c}-\underline{c})^2}{\underline{c}^2} < \frac{8}{27}$, or $\overline{\theta}$ in the case that $\frac{8}{27} \leq \frac{(\bar{c}-\underline{c})^2}{\underline{c}^2} < \frac{8}{9}$), then private firm's transferability is pro-competitive since it lowers expected monetary transfer to the public good provider. However, when θ is higher than the threshold, then private firm's transferability is counter-competitive since it increases the expected monetary transfer to the public good provider.

The result presented in this section applies to the case which the public good was previously provided by a public firm (i.e., civil servants) and the public authority allows for competition between public and private firms. Proposition 3 shows that consumers' gains from changing to private ownership are high in industries where private firms have low ability on transferring knowledge among different markets.

From Proposition 3, we can derive the following testable implication which relates the expected monetary transfer to ownership:

Implication 2 *The expected second-period transfer under private ownership is higher than the second-period expected transfer under public ownership.*

Implication 2 comes from the existence of private firm's transferability. Private firm's transferability bounds the expected transfers to incumbent private firms, whereas increases the expected transfers to the private firms.

It is consistent with Bontemps, Martimort, Roucolle and Thomas (2009). They analyze the regulated price of potable water in France and show that prices of water in cities with private ownership are higher on average than in cities with public ownership.

Comparison between the first and second period expected transfers. Taking Equations (31) and (32), we can compare the first to the second-period expected transfer. The implication below describes the dynamics of the expected transfer in the model.

Implication 3 *The first-period expected transfer to the public good provider is lower than the second-period expected transfer.*

Implication 3 comes also from the existence of learning-by-doing. The intuition for this result is the following: Learning-by-doing gives the incumbent firm an advantage over the entrant in a subsequent competition, which translates into higher probability of winning and higher profit (higher market power), and high public monetary transfer for the second-period public good provider. However, competing firms anticipate these benefits of being incumbent. Hence, they fiercely compete for the first-period competition, producing low first-period profit for firms and, therefore, low public monetary transfer for the first-period public good provider.

It is consistent with Gagnepain, Ivaldi and Martimort (2009) who analyze the public subsidies to providers of local public transportation in France. They show that subsidies to public and private firms have been increasing overtime. It is also consistent with Shaoul (1997) who investigating the privatized firms in Water and Sewerage Companies of England and Wales. Shaoul (1997) finds that the prices charged by private firms in their second period contract of water provision is substantially higher than the prices charged by private firms in their first period contract. These empirical implications are also consistent with the evidence surveyed by Renzetti and Dupont (2004).

4.2 Ex-ante Consumers Welfare and Transferability

In this section, we analyzed the effect of private firm's transferability θ on the total consumers welfare. As defined in Equation (1), the total ex-ante consumers welfare corresponds to the sum of the first-period and second-period expected consumers welfare. We can compute the total ex-ante consumers welfare in equilibrium, replacing (31) and (32) in (1).

The proposition below shows that private firm's transferability decreases the total ex-ante expected consumers welfare.

Proposition 4 *If* $\frac{(\bar{c}-\underline{c})^2}{\underline{c}^2} > \frac{4}{9}$*, then the equilibrium total ex-ante expected consumers welfare decreases with private firm's transferability* θ*:* $\frac{\partial W}{\partial \theta} < 0$.

Proposition 4 claims that if variance of first-period production cost is relatively high, $\frac{(\bar{c}-\underline{c})^2}{\underline{c}^2} > \frac{4}{9}$, then the pro-competitive effect of transferability on the ex-ante second-period consumers welfare is off set by the counter-competitive effect of transferability on the ex-ante first-period consumers welfare.

5 Discussion

5.1 Policy Recommendation

In most of the auctions analyzed in this paper, public and private firms are asymmetric competitors.[12] In the first-period auction, for instance, the private firm has higher benefit of winning the auction than the public firm, what makes the private firm more aggressive than its public opponent. In the second-period auction, an asymmetry is also present: The incumbent private firm is more likely to have lower expected production cost than the entrant public firm, and the incumbent public firm is more likely to have lower expected production cost than the entrant private firm without transferability.

Under these circumstances, one may wonder if there is a manner how to optimally structure the public-private competition that takes into account the existing asymmetries between competitors, in order to minimize the monetary transfer from the public authority to the public good provider. The literature on mechanism design and auction shows that there are other mechanisms than the standard first-price procurement auction that leads to lower monetary transfer. In particular, McAfee and and McMillan (1989) shows that a first-price procurement auction which favors the less efficient firm (bidder) leads to lower cost of procurement than the standard one.

Given these results, a strategic and benevolent local public authority should design a sequential auction in certain way that the consumers welfare is maximized. Applying McAfee and and McMillan to our framework, we can show that the optimal sequential biased auction would be the one which favors the public firm in the first-period auction and the entrant public firm in the second-period auction. In the case that the private firm is an entrant without transferability (i.e., without outside learning), it should also be favored in the second-period auction.[13] These results should be interpreted as a policy recommendation that may be applied to the industries which there is low degree of competition and learning-by-doing.

The literature has suggested other possible solutions for the lack of competition in the provision of public utilities. For instance, Webb and Ehrhardt (1998), a policy paper on how to structure competition in the water sector, claim that the introduction of competition through competing networks (competing suppliers each establish their own distribution system), the retail competition (entrants purchase bulk water supply from the incumbent and construct its own distribution network) and common carriage competition (several water utilities use a

[12]In fact, in the first-period auction and in all auctions in the second period, except the one in the state $X = 2$, firms are asymmetric.

[13]In the case that the public authority wants to maximize the social welfare rather the consumers welfare, the optimal sequential biased auction will be the one which favors the private firm in the first-period auction and the incumbent firm in the second-period auction.

single network to supply customers, and customers can choose their water supplier) overcome the abusive use of monopoly power in water services.

Unfortunately, the introduction of competition networks and retail competition seems not to provide the desired solution given the natural monopoly feature of water provision: the costs of installing competing networks are extremely high. Common carriage competition clearly overcomes the lack of competition in the retail market, however it keeps the monopoly power in the network distribution. As it turns out, the upstream firm, which operates the network distribution - an essential facility, may also have learning-by-doing. In that case, the argument in that paper, which explains the lack of competition in provision of local utilities, also applies to the competition for whom operates the network distribution facility. Therefore, firms in retail market will be charged a high fixed fee to access the network distribution, and, consequently, consumers will end up paying high price for consumption of water.

Another potential solution is the organization of a Public-Private Partnership (PPP) to provide public services. In a PPP, partners (a government and a private firm) cooperate in the provision of public good, and share information and knowledge about state of the network.[14] As a result, the public entrant firm will be as efficient as the private incumbent one, which has learning-by-doing. However, it does not seem to be credible that an private incumbent firm will share such valuable information with a potential future public competitor. Therefore, this solution does not seem to be appropriated to deal with the lack of competition.

5.2 Big Firms versus Small Firms: U.S. Small Business Act

Analyzing the competition between public and private firms in a competition for public services, the key difference between firms is that private firms serve and compete in several markets, whereas local public ones only provide the local public good in the local city. As it turns out, we could relabel firm's name and call private firm as a global (or big) firm, and public firm as local (or small) firm. In this case, the policy recommendation described above would be the following: government should design biased procurement auctions which favor small firm leads in order to increase consumers welfare.

This policy recommendation is consistent with the U.S. Small Business Act (SBA). The SBA was established on July 30, 1953, by the United States Congress with the passage of the Small Business Act, stipulates a "fair proportion" of government contracts and sales of surplus property to small business. That can be understood as favoritism to small firms vis-à-vis big firms. Consistently with this interpretation of the SBA, Marion (2007) documents that in California auctions for road construction contracts, small businesses receive a 5-percent bid

[14]See Moszoro and Gasiorowski (2008) for the optimal capital structure of Public-Private Partnerships with exchange of knowledge.

preference in auctions for projects using only state funds and no preferential treatment on projects using federal aid.

Denes (1997), analyzing the federal dredging contrast supplied by the U.S. Army Corps of Engineers, shows that the Small Business Act may reduce the cost of contracted services as long as the pool of bidders is not reduced. It is consistent with the policy recommendation described above.

Favoritism to small businesses is not only adopted in the U.S.,[15] for instance, Nakabayashi (2009) documents that half of the Japanese public construction procurement contracts is set aside for small and medium enterprises. Nakabayashi shows that the small business set-aside programs reduces government procurement costs because it induces competition.

6 Conclusion

In this paper we explain why the introduction of competition among potential providers for provision of local public services does not seem to produce the desired and expected increase in consumers welfare. We show that the existence of learning-by-doing in the provision of public services and private firms' ability on transferring learning inside the firm may explain the empirical evidence.

In addition, the paper argues that a biased procurement auction in the first period, in favor of local public firm (less efficient), helps to extract the rent of the private one (more efficient) and leads to higher consumers welfare.

A potential interesting extension of this paper is to describe the optimal biased sequential procurement auction. Another extension which may provide interesting results is to introduce more private firms in the model, allowing for competition between private firms. Finally, we assume that the probability θ is an exogenous variable. It can be endogenized in an extended model in which there are several cities, each one has its own public firm, and the private firm can provide the public good in all cities. In this suggested extension, the probability θ will be determined in equilibrium.

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Appendix

Proof of Lemma 1

(i): $s_2^{*G}(c, X=1) - s_2^{*F}(c, X=1) = \frac{2\overline{c}+\widetilde{c}}{6} + \frac{1}{2}c - \frac{2\widetilde{c}+\overline{c}}{6} - \frac{1}{2}c = \overline{c} - \widetilde{c} > 0$.
(ii): Public probability of winning is defined as $\tau_2^G = Prob(s_2^{*G} < s_2^{*F})$. Replacing (11) and (12) in this probability, we obtain that

$$\tau_2^G = Prob(c_2^F - c_2^G > \frac{\overline{c} - \widetilde{c}}{3})$$

Define the random variable $y = c_2^F - c_2^G$. Since $c_2^F \sim U[\underline{c}, \overline{c}]$ and $c_2^G \sim U[0, \widetilde{c}]$. Then, y has the following cumulative distribution function:

$$F(y)=\begin{cases}0 & ,y\leq \underline{c}-\widetilde{c}\\ \frac{(\widetilde{c}+y-\underline{c})(y+\widetilde{c}-\underline{c})}{2(\overline{c}-\underline{c})\widetilde{c}} & ,\underline{c}-\widetilde{c}\leq y<\overline{c}-\widetilde{c}\\ 1-\frac{(\overline{c}-y)(y+\widetilde{c}-\underline{c})}{2(\overline{c}-\underline{c})\widetilde{c}} & ,\overline{c}-\widetilde{c}\leq y<\overline{c}\\ 1 & ,y>\overline{c}\end{cases}$$

Note that $Prob(y=c_2^F-c_2^G\leq \overline{c}-\widetilde{c})=F(\underline{c}-\widetilde{c})=\frac{1}{2}$. Because $\frac{\overline{c}-\widetilde{c}}{3}<\overline{c}-\widetilde{c}$ and $F(y)$ is increasing in y, then $Prob(y=c_2^F-c_2^G\leq\frac{\overline{c}-\widetilde{c}}{3})=F(\frac{\overline{c}-\widetilde{c}}{3})<\frac{1}{2}$. Note that $\tau_2^G=1-Prob(c_2^F-c_2^G\leq\frac{\overline{c}-\widetilde{c}}{3})=1-F(\frac{\overline{c}-\widetilde{c}}{3})>\frac{1}{2}$.

In particular, $F(\frac{\overline{c}-\widetilde{c}}{3})=\frac{(2\widetilde{c}-3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$. Since $\tau_2^G=1-F(\frac{\overline{c}-\widetilde{c}}{3})$, then $\tau_2^G=1-\frac{(2\widetilde{c}-3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$. Therefore, $\tau_2^F=\frac{(2\widetilde{c}-3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$. □

Proof of Lemma 2

(i): $s_2^{*G}(c,X=2)-s_2^{*F}(c,X=2)=\frac{\widetilde{c}}{2}+\frac{1}{2}c-\frac{\widetilde{c}}{2}-\frac{1}{2}c=0$
(ii): Public probability of winning is defined as $\tau_2^G=Prob(s_2^{*G}<s_2^{*F})$. Replacing (16) in this probability, we obtain that

$$\tau_2^G=Prob(c_2^F-c_2^G>0)$$

Define the random variable $y=c_2^F-c_2^G$. Since $c_2^F\sim U[0,\widetilde{c}]$ and $c_2^G\sim U[0,\widetilde{c}]$. Then, y has the following cumulative distribution function:

$$F(y)=\begin{cases}0 & ,y\leq -\widetilde{c}\\ \frac{(y+\widetilde{c})^2}{2\widetilde{c}^2} & ,-\widetilde{c}<y\leq 0\\ \frac{1}{2} & ,y=0\\ 1-\frac{(y-\widetilde{c})^2}{2\widetilde{c}^2} & ,0<y\leq\widetilde{c}\\ 1 & ,y>\widetilde{c}\end{cases}$$

Note that $Prob(y=c_2^F-c_2^G\leq 0)=F(0)=\frac{1}{2}$. Note that $\tau_2^G=1-Prob(c_2^F-c_2^G\leq 0)=1-F(0)=\frac{1}{2}$. Therefore, $\tau_2^F=\frac{1}{2}$. □

Proof of Lemma 3

The proof is similar to the proof of Lemma 1. □

Proof of Lemma 4

We will do it in two steps. We first characterize firms' second-period expected equilibrium payoff before the nature draws second-period costs, i.e., date *v*. Then, we turn to characterization of the expected equilibrium payoffs before the nature draws private firm's transferability, i.e., date *iv*.

Continuation Payoffs: When Nature draws second-period costs

In the previous subsection, we characterized the equilibrium expected payoffs in the all second-period auctions. At that stage, firms knew their own production cost. However, in this subsection the aim it to characterize firms' expected payoff when firms do not know their own production cost, i.e., at date v, when firms know already in which second-period contingency they are, but the nature has not drawn yet second-period production costs. At this stage, firms will infer the second-period payoff basing on the available information: the contingency at the second-period, represented by the state variable X.

Let us describe the expected equilibrium payoff in all possible states X in the second period.

■ **State $X = 1$.** At this contingency and after the nature draws the production costs, firm's equilibrium profits are, respectively, characterizes by equations (13) and (14). However, we would like to characterize the equilibrium expected payoffs before the nature draws second-period costs. In order to do so, let us first describe the private firm's expected payoff at date v, and the we turn to the public one.

The private firm infers its second-period profit taking the expectation of (13) over c_2^F. Because in the State $X = 1$, the private firm's second-period cost c_2^F will be drawn an uniform distribution on $[\underline{c}, \overline{c}]$, then its expected payoff will be equal to

$$E[\pi_2^{*F}(c_2^F, X = 1)] = \int_{\overline{c}}^{\underline{c}} \frac{1}{2\widetilde{c}(\overline{c} - \underline{c})} \Big[\frac{2\widetilde{c} + \overline{c}}{3} - c_2^F\Big]^2 dc_2^F = \underline{\Pi}, \tag{33}$$

with $\underline{\Pi} := \frac{4\widetilde{c}^2+\overline{c}^2-2\overline{c}\widetilde{c}+3\underline{c}^2-6\overline{c}^2}{18\widetilde{c}}$.

Let us now turn to the description of public firm's expected profit. Similarly, the public firms computes its expected payoff taking the expectation of (14) over c_2^G. Since the public firm's second-period cost c_2^G will be distributed according to the uniform distribution on $[0, \widetilde{c}]$ in the state $X = 1$, then its expected payoff will be

$$E[\pi_2^{*G}(c_2^G, X = 1)] = \int_{0}^{\widetilde{c}} \frac{1}{2\widetilde{c}(\overline{c} - \underline{c})} \Big[\frac{2\overline{c} + \widetilde{c}}{3} - c_2^F\Big]^2 dc_2^F = \overline{\Pi}, \tag{34}$$

with $\overline{\Pi} := \frac{4\overline{c}^2+\widetilde{c}^2-2\overline{c}^2}{18\widetilde{c}}$.

The following Lemma compares incumbent public firm and entrant (and without external learning) private firm's second-period equilibrium expected profit.

Lemma 6 *The second-period expected profit of the incumbent public firm is strictly higher than the one of the entrant private firm:* $E[\pi_2^{*G}(c_2^G, X = 1) > E[\pi_2^{*F}(c_2^F, X = 1)]$.

Proof of Lemma 6 $\overline{\Pi} - \underline{\Pi} = \frac{4\overline{c}^2+\widetilde{c}^2-2\overline{c}^2}{18\widetilde{c}} - \frac{4\widetilde{c}^2+\overline{c}^2-2\overline{c}\widetilde{c}+3\underline{c}^2-6\overline{c}^2}{18\widetilde{c}} = \frac{\overline{c}^2-\widetilde{c}^2-\underline{c}^2+2\widetilde{c}\underline{c}}{18\widetilde{c}} = \frac{4\underline{c}(\overline{c}-\underline{c})}{18\widetilde{c}} > 0$ because we assumed that $\widetilde{c} = \overline{c} - \underline{c}$. □

The result of Lemma 6 is intuitive since the public firm expects to have lower cost and also is less aggressive than the private one.

■ **State** $X = 2$**.** To compute firms' expected profit before the nature draws the second-period cost, we need to compute the expectation of (17) over c_2^i, with $i = F, G$. Because in the State $X = 2$ both firms have their second-period cost c_2^i drawn from an uniform distribution on $[0, \widetilde{c}]$, then firm-i's expected payoff will be

$$E[\pi_2^{*i}(c_2^i, X = 2)] = \int_{\widetilde{c}}^{0} \frac{(\widetilde{c} - c_2^F)^2}{2\widetilde{c}^2} dc_2^i = \Pi^C, \tag{35}$$

with $\Pi^C := \frac{\widetilde{c}}{6}$ and $i = \{F, G\}$.

■ **State** $X = 3$**.** As before, we can derived firms' second-period expected profit for the state $X = 3$ using the results for the case which $X = 1$, just replacing the private firm's expected profit for the public one, and vice-versa. Hence, private and public firm's expected second-period profit are, respectively, described by

$$E[\pi_2^{*F}(c_2^F, X = 3)] = \overline{\Pi}, \tag{36}$$

$$E[\pi_2^{*G}(c_2^G, X = 3)] = \underline{\Pi}. \tag{37}$$

Similarly to the state $X = 1$, the incumbent private firm has higher expected payoff than the entrant public one, $\overline{\Pi} > \underline{\Pi}$.

After some algebraic manipulations, we obtain that:

$$\overline{\Pi} - \Pi^C > \Pi^C - \underline{\Pi}. \tag{38}$$

Equation 38 says that the increase in profit for being stronger than its competitor $\overline{\Pi} - \Pi^C$ is higher than the increase in profit for becoming symmetric to its competitor $\Pi^C - \underline{\Pi}$.

Continuation Payoffs: When Nature draws private firm's transferability

In the previous subsection, we described firms' second-period expected equilibrium payoffs before the nature draws second-period costs, i.e., date v. Now, let us characterize the second-period expected equilibrium profits before the nature draws private firm's transferability (date iv). Since private transferability is unknown at date iv, firms will compute expected profit taking into account that with probability θ, the private firm has transferability (outside learning), and with probability $1 - \theta$, the private firm does not.

Similarly to the previous section, the expected profit of the firms depends on which firm was the first-period public good provider. For this reason, we first analyze the firms' expected payoff when the private firm was the first-period public good provider, then we turn to the opposite case which the public firm was the first-period public good provider.

The private firm wins the first-period auction in the city when private firm's first-period bid is lower the public firm's first-period bid ($s_1^F < s_1^G$). In this circumstance, certainly the second-period state will be $X = 3$ because the private firm will be incumbent and the public firm will be entrant. Therefore, only the private firm will have lower expected second-period cost. Hence, as described in the previous subsection, when $X = 3$ public and private firm's second-period expected profit are respectively described by

$$\Pi_2^F(s_1^F < s_1^G) = \overline{\Pi}$$

and

$$\Pi_2^G(s_1^F < s_1^G) = \underline{\Pi}$$

Yet when public firm's first-period bid is lower the private firm's first-period bid ($s_1^F > s_1^G$), the public firm wins the first-period auction in the city. When it occurs, there are two possible states in the second-period. With probability θ, private firm will be have transferability (incumbent elsewhere). In this case, the second-period contingency will be $X = 2$. Yet with probability $1 - \theta$, private firm will not have transferability. Consequently, the second-period contingency will be $X = 1$.

Let us first describe the private firm's payoff and then turn to the characterization of the public one. With probability θ, the second-period state will be $X = 2$. Hence, as described in equation (35), the private firm's expected payoff is Π^C. Yet with probability $1 - \theta$, the second-period sate will be the $X = 1$. Hence, as described in equation (37), the private firm's expected payoff is $\underline{\Pi}$. Therefore, the private firm's payoff after the public firm winning the first-period auction is:

$$\Pi_2^F(s_1^F > s_1^G) = \theta\Pi^C + (1 - \theta)\underline{\Pi}.$$

Let us now turn to the public firm's payoff after its first-period of public good provision. Similarly, with probability θ, the second-period state will be $X = 2$. Hence, as described in equation (35), public firm's expected payoff is Π^C. Yet with probability $1 - \theta$, the second-period state will be $X = 1$. Hence, as described in equation (36), the public firm's expected payoff is $\overline{\Pi}$. Thus, public firm's second-period expected payoff after it wins the first-period auction will be:

$$\Pi_2^G(s_1^F > s_1^G) = \theta\Pi^C + (1 - \theta)\overline{\Pi}.$$

Proof of Proposition 1

(i): $s_1^{*G}(c) - s_1^{*F}(c) = \frac{\bar{c}}{2} + \frac{-\theta\Pi^C + \underline{\Pi}(3-\theta) - \overline{\Pi}(3-2\theta)}{2} + \frac{1}{2}c - \frac{\bar{c}}{2} - \frac{\theta\Pi^C + \underline{\Pi}(3-2\theta) - \overline{\Pi}(3-\theta)}{2} - \frac{1}{2}c$. After some algebraic manipulations, we obtain that $s_1^{*G}(c) - s_1^{*F}(c) = \frac{\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)$. From equation (38), we know that $\overline{\Pi} + \underline{\Pi} - 2\Pi^C > 0$. Therefore, it follows that $s_1^{*G}(c) - s_1^{*F}(c) > 0$.
(ii): Public probability of winning is defined as $\tau_1^G = Prob(s_1^{*G} < s_1^{*F})$. Replacing (30) and (29) in this probability, we obtain that

$$\tau_1^G = Prob(c_1^F - c_1^G > \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C))$$

Define the random variable $y = c_1^F - c_1^G$. Since $c_1^F \sim U[\underline{c}, \overline{c}]$ and $c_1^G \sim U[\underline{c}, \overline{c}]$. Then, y has the following cumulative distribution function:

$$F(y) = \begin{cases} 0 & , y \leq \underline{c} - \overline{c} \\ \frac{(\overline{c}-\underline{c}+y)^2}{2(\overline{c}-\underline{c})^2} & , \underline{c} - \overline{c} \leq y < 0 \\ 1 - \frac{(\overline{c}-\underline{c}-y)^2}{2(\overline{c}-\underline{c})^2} & , 0 \leq y < \overline{c} - \underline{c} \\ 1 & , y > \overline{c} - \underline{c} \end{cases}$$

Hence,

$$\tau_1^G = 1 - F\Big(\frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big)$$

Note that if $\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C) < \overline{c}-\underline{c}$, then $F(\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)) < 1$, and therefore, $\tau_1^G > 0$. Note that $\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C) < \frac{2}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)$. Therefore, if $\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C) < \overline{c}-\underline{c}$, then $F(\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)) < 1$, and therefore, $\tau_1^G > 0$. Because $\frac{2}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C) = \frac{8\underline{c}^2}{18(\overline{c}-\underline{c})}$, then if $\frac{8\underline{c}^2}{18(\overline{c}-\underline{c})} < \overline{c}-\underline{c}$ we have that $\tau_1^G = 1 - F(\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)) > 0$. After some algebraic manipulations, that inequality is equivalent to $(\overline{c}-\underline{c})^2 - \frac{8\underline{c}^2}{27} > 0$. Therefore, if $(\overline{c}-\underline{c})^2 - \frac{8\underline{c}^2}{27} > 0$, then $F(\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)) < 1$ and $\tau_1^G > 0$. In particular, $F(\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)) = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c}-\underline{c}-\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)\Big]$. Because $\tau_1^F = F(\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C))$ and $\tau_1^G = 1 - F(\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)$, then $\tau_1^F = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c}-\underline{c}-\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C))\Big]$, and $\tau_1^G = \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c}-\underline{c}-\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)\Big]$.

Yet $(\overline{c}-\underline{c})^2 - \frac{8\underline{c}^2}{27} \leq 0$, there exists $\widehat{\theta} \in (0,1)$, defined as $\frac{3(\overline{c}-\underline{c})}{2(\overline{\Pi}+\underline{\Pi}-2\Pi^C)}$ such that for $\theta \geq \widehat{\theta}$, $\tau_1^F = 1$ and $\tau_1^G = 1$; and for $\theta < \widehat{\theta}$, $\tau_1^F = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c}-\underline{c}-\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)\Big]$ and $\tau_1^G = \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c}-\underline{c}-\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)\Big]$. To show it, note that if $(\overline{c}-\underline{c})^2 - \frac{8\underline{c}^2}{27} \leq 0$ and $\theta = 1$, then $\frac{2}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C) > \overline{c}-\underline{c}$. Yet when $\theta = 0$ we have that $\frac{2}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C) < \overline{c}-\underline{c}$. Because $\frac{2}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)$ is monotonically increasing in that θ, then there $\widehat{\theta} \in (0,1)$ such that for $\theta \geq \widehat{\theta}$, then $\frac{2}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C) > \overline{c}-\underline{c}$, which implies that $\tau_1^F = 1$ and $\tau_1^G = 1$. Yet when $\theta < \widehat{\theta}$, then $\frac{2}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C) \geq \overline{c}-\underline{c}$, which implies that $\tau_1^F = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c}-\underline{c}-\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)\Big]$ and $\tau_1^G = \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c}-\underline{c}-\frac{2\theta}{3}(\overline{\Pi}+\underline{\Pi}-2\Pi^C)\Big]$. □

Proof of Proposition 2

(i): From equation (30), we know that $s_1^{*G}(\theta, c_1^G) = \frac{\overline{c}}{2} + \frac{-\theta\Pi^C+\underline{\Pi}(3-\theta)-\overline{\Pi}(3-2\theta)}{2} + \frac{1}{2}c_1^G$. Deriving this expression with respect to θ, we obtain that $\frac{\partial s_1^{*G}(.)}{\partial\theta} = \frac{\overline{\Pi}-\Pi^C+\overline{\Pi}-\underline{\Pi}}{3}$. We will show that $\overline{\Pi}-\Pi^C > 0$and $\overline{\Pi}-\underline{\Pi} > 0$.

Let us first show that $\overline{\Pi}-\Pi^C > 0$. From (34) and (35), we find that $\overline{\Pi}-\Pi^C = \frac{1}{18(\overline{c}-\underline{c})}(4\overline{c}^2+\widetilde{c}^2 - 2\underline{c}\widetilde{c} - 3\widetilde{c}^2)$. Because $\widetilde{c} = \overline{c}-\underline{c}$, then $\overline{\Pi}-\Pi^C = \frac{\underline{c}}{18(\overline{c}-\underline{c})}(3\overline{c}-\underline{c})$, which is positive by the assumption that $\overline{c} > \underline{c}$.

Let us now show that $\overline{\Pi}-\underline{\Pi} > 0$. From (34) and (33), we find that $\overline{\Pi}-\underline{\Pi} = \frac{4\underline{c}(\overline{c}-\underline{c})}{18(\overline{c}-\underline{c})} > 0$.

(ii): From equation (29), we know that $s_1^{*F}(\theta, c_1^F) = \frac{\overline{c}}{2} + \frac{\theta\Pi^C+\underline{\Pi}(3-2\theta)-\overline{\Pi}(3-\theta)}{2} + \frac{1}{2}c_1^F$. Deriving this

expression with respect to θ, we obtain that $\frac{\partial s_1^{*F}(.)}{\partial\theta} = \frac{\overline{\Pi}-\underline{\Pi}+\Pi^C-\underline{\Pi}}{3}$.

From (34) and (33), we find that $\overline{\Pi} - \underline{\Pi} = \frac{4\underline{c}(\overline{c}-\underline{c})}{18(\overline{c}-\underline{c})}$. From (35) and (33),we find that $\Pi^C - \underline{\Pi} = \frac{2\underline{c}(3\widetilde{c}-2\underline{c})}{18(\overline{c}-\underline{c})}$. Given these expressions and taking into account that $\widetilde{c} = \overline{c} - \underline{c}$, we obtain that $\frac{\partial s_1^{*F}(.)}{\partial\theta} = \frac{\overline{\Pi}-\underline{\Pi}+\Pi^C-\underline{\Pi}}{3} = \frac{\underline{c}}{9(\overline{c}-\underline{c})}(5\overline{c} - 7\underline{c})$. That expression is positive if and only if $\overline{c} \geq \frac{7\underline{c}}{5} = 1.4\underline{c}$.

(iii): From Proposition 1, we know that if $(\overline{c} - \underline{c})^2 - \frac{8\underline{c}^2}{27} < 0$, then there exists $\widehat{\theta}$ such that for $\theta \geq \widehat{\theta}$, $\tau_1^F = 1$ and $\tau_1^G = 1$; and for $\theta < \widehat{\theta}$, $\tau_1^F = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$ and $\tau_1^G = \frac{1}{2(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$. In the case that $(\overline{c}-\underline{c})^2 - \frac{8\underline{c}^2}{27} < 0$ and $\theta \geq \widehat{\theta}$, $\frac{\partial\tau_1^F}{\partial\theta} = \frac{\partial\tau_1^G}{\partial\theta} = 0$. Yet in the case that $(\overline{c}-\underline{c})^2 - \frac{8\underline{c}^2}{27} < 0$ and $\theta < \widehat{\theta}$, $\frac{\partial\tau_1^F}{\partial\theta} = \frac{2(\overline{\Pi}+\underline{\Pi}-2\Pi^C)}{3(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$, which is positive since that $\overline{c} - \underline{c} \geq \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C) > 0$ for $\theta < \widehat{\theta}$. And $\frac{\partial\tau_1^G}{\partial\theta} = -\frac{2(\overline{\Pi}+\underline{\Pi}-2\Pi^C)}{3(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$, which is also negative since that $\overline{c} - \underline{c} \geq \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C) > 0$ for $\theta < \widehat{\theta}$.

Also from Proposition 1, we know that when $(\overline{c} - \underline{c})^2 - \frac{8\underline{c}^2}{27} > 0$, $\tau_1^F = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$ and $\tau_1^G = \frac{1}{2(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$. In the case that $\frac{\partial\tau_1^F}{\partial\theta} = \frac{2(\overline{\Pi}+\underline{\Pi}-2\Pi^C)}{3(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$, which is positive since that $\overline{c} - \underline{c} \geq \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C) > 0$ for all θ . And $\frac{\partial\tau_1^G}{\partial\theta} = -\frac{2(\overline{\Pi}+\underline{\Pi}-2\Pi^C)}{3(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$, which is also negative since that $\overline{c} - \underline{c} \geq \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C) > 0$ for all θ. □

Proof of Lemma 5

From Equation (31), we know that $E[p_1] = \tau_1^G E[s_1^{*G}] + \tau_1^F E[s_1^{*F}]$. Deriving it with respect to θ, we obtain

$$\frac{\partial E[p_1]}{\partial\theta} = -\frac{\partial\tau_1^F}{\partial\theta}[E[s_1^{*G}] - E[s_1^{*F}]] + \tau_1^G\frac{\partial E[s_1^{*G}]}{\partial\theta} + \tau_1^F\frac{\partial E[s_1^{*F}]}{\partial\theta}, \tag{39}$$

since that $\frac{\partial\tau_1^G}{\partial\theta} = -\frac{\partial\tau_1^F}{\partial\theta}$. Note that, $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} \geq \frac{4}{9}$ implies $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} \geq \frac{8}{27}$. Hence, from Proposition 1, we have that $\frac{\partial\tau_1^F}{\partial\theta} > 0$ because $\tau_1^F = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$ and $\tau_1^G = \frac{1}{2(\overline{c}-\underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]$.

To compute $\frac{\partial E[p_1]}{\partial\theta}$, let us replace in (39) the value of τ_1^F, τ_1^G, which were defined in Proposition 1, $E[s_1^{*F}]$ and $E[s_1^{*G}]$, defined in equations (29) and (30), and $\frac{\partial\tau_1^F}{\partial\theta}$ and $\frac{\partial\tau_1^G}{\partial\theta}$, which were defined in Proposition 2 , we obtain that

$$\begin{aligned}\frac{\partial E[p_1]}{\partial\theta} \quad = \quad & -\frac{2(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)}{3(\overline{c} - \underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]\frac{\theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)}{3} \\ & + \frac{1}{2(\overline{c} - \underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]^2\frac{2\overline{\Pi} - \underline{\Pi} - \Pi^C}{3} \\ & + \left[1 - \frac{1}{2(\overline{c} - \underline{c})^2}\left[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\right]^2\right]\frac{2\overline{\Pi} + \Pi^C - 2\underline{\Pi}}{3}.\end{aligned}$$

Note that $\frac{\partial E[p_1]}{\partial\theta}|_{\theta=0} = \frac{1}{6}(2\overline{\Pi} - \Pi^C - \underline{\Pi}) + \frac{1}{6}(\overline{\Pi} + \Pi^C - 2\underline{\Pi})$. The first term is always positive, by equation (38). By Proposition 2, we know that the second term is positive if $\overline{c} > \frac{7\underline{c}}{5}$. Because we have assumed that $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} \geq \frac{4}{9}$, which implies that $\overline{c} \geq \frac{5\underline{c}}{3}$. Consequently,$\overline{c} > \frac{7\underline{c}}{5}$ Therefore, we have that $\frac{\partial E[p_1]}{\partial \theta}|_{\theta=0} > 0$.

We will show that $\frac{\partial E[p_1]}{\partial \theta}$ is monotonically increasing in θ. To show it, we will derive expression (48) with respect to θ.

$$\frac{\partial^2 E[p_1]}{\partial \theta^2} = -\frac{4}{9}\frac{(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)^2}{(\overline{c} - \underline{c})^2}\Big[\overline{c} - \underline{c} - \theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big] \tag{40}$$

If $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} \geq \frac{4}{9}$, then $\overline{c} - \underline{c} \geq (\overline{\Pi} + \underline{\Pi} - 2\Pi^C)$, which implies that $\overline{c} - \underline{c} - \theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C) \geq 0$. Hence, $\frac{\partial^2 E[p_1]}{\partial \theta^2} > 0$.

Hence, if $\frac{\partial^2 E[p_1]}{\partial \theta^2} > 0$ and $\frac{\partial E[p_1]}{\partial \theta}|_{\theta=0} > 0$, then $\frac{\partial E[p_1]}{\partial \theta} > 0$ for all θ. □

Second-Period Expected Transfer

We first compute the second-period expected transfer in each of these states, and then we compute the ex-ante second-period expected transfer, $E[p_2]$, as the expectation over all states

■ **State $X = 1$.** As described in Section 3.1, state $X = 1$ is the second-period state which the public firm was the first-period public good provider in the city and the private firm was not incumbent elsewhere. In this state, the public firm expects to have lower second-period cost, whereas the private firm does not. The private and public firm's equilibrium bidding strategies in this state are, respectively, s_2^{*F} and s_2^{*G}, described in (11) and (12).

With probability τ_2^G, defined in Lemma 1, the public firm wins the second-period auction. Hence, the local public authority expects to pay to public firm its bid s_2^{*G}, defined in (12). Yet with probability τ_2^F, also defined in Lemma 1, the private firm wins the second-period auction. In that case, the local public authority expects to pay to the private firm its bid s_2^{*F}, defined in (11). Hence, second-period expected transfer in the State $X = 1$ is given by the following expression:

$$E[p_2|X = 1] = \tau_2^G(X = 1)E[s_2^{*G}(X = 1)] + \tau_2^F(X = 1)E[s_2^{*G}(X = 1)]. \tag{41}$$

Replacing in the expression above the equilibrium strategies, (11) and (12), and firm's probabilities of winning, defined in defined in Lemma 1, we obtain the following second-period expected transfer in State $X = 1$:

$$E[p_2|X = 1] = \overline{p}. \tag{42}$$

where $\overline{p}$ is defined as follows

$$\overline{p} = \frac{4\overline{c} + 5\widetilde{c}}{12}\Big[\frac{18\widetilde{c}(\overline{c} - \underline{c}) - (2\widetilde{c} + 3\underline{c} + \overline{c})^2}{18\widetilde{c}(\overline{c} - \underline{c})}\Big] + \frac{4\widetilde{c} + 5\overline{c} + 3\underline{c}}{12}\Big[\frac{(2\widetilde{c} + 3\underline{c} + \overline{c})^2}{18\widetilde{c}(\overline{c} - \underline{c})}\Big] \tag{43}$$

■ **State $X = 2$.** At the second-period contingency which $X = 2$, the public firm was the first-period public good provider in the city and the private firm is incumbent elsewhere, therefore it has outside learning. Hence, firms are perfectly symmetric and have the same equilibrium expected payoffs before the nature draws second-period costs. As described in Section 3.1 state $X = 2$, private and public firm's equilibrium bidding strategies are, respectively, s_2^{*F} and s_2^{*G}, described in (16).

The second-period expected transfer in the state $X = 2$ can also be expressed by equation (41), after replacing $X = 1$ for $X = 2$. After that, replace the equilibrium strategies s_2^{*F} and s_2^{*G} described in (16), and the firm's probabilities of winning τ_2^{*F} and τ_2^{*G}, defined in Lemma 2, in that expression, we obtain the following second-period expected transfer in the state $X = 2$:

$$E[p_2|X = 1] = \underline{p}. \tag{44}$$

where $\underline{p}$ is defined as follows

$$\underline{p} = \frac{3\widetilde{c}}{4} \tag{45}$$

The following lemma compares the second-period expected transfer in the state $X = 2$ with the second-period expected transfer in the state $X = 1$.

Lemma 7 *The second-period expected transfer in the state $X = 2$, described by $\underline{p}$, is lower than the second-period expected transfer in State $X = 1$, described by $\overline{p}$,*

$$\underline{p} < \overline{p}.$$

Proof of Lemma 7. From equation (43), we have that

$$\overline{p} = \frac{4\overline{c} + 5\widetilde{c}}{12}\left[\frac{18\widetilde{c}(\overline{c} - \underline{c}) - (2\widetilde{c} + 3\underline{c} + \overline{c})^2}{18\widetilde{c}(\overline{c} - \underline{c})}\right] + \frac{4\widetilde{c} + 5\overline{c} + 3\underline{c}}{12}\left[\frac{(2\widetilde{c} + 3\underline{c} + \overline{c})^2}{18\widetilde{c}(\overline{c} - \underline{c})}\right].$$

and from equation (45), we know that

$$\underline{p} = \frac{3\widetilde{c}}{4}$$

Let us proof Lemma 7 in four steps. Step (i): From Lemma 1, we know that $\frac{18\widetilde{c}(\overline{c}-\underline{c})-(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})} > \frac{1}{2} > \frac{(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$. Step (ii): Doing some algebraic manipulations, it is easy to see that $\frac{4\overline{c}+5\widetilde{c}}{12} < \frac{4\widetilde{c}+5\overline{c}+3\underline{c}}{12}$ if $\widetilde{c} < \overline{c} + 3\underline{c}$. Because, $\overline{c} > \widetilde{c}$, then that inequality holds. Step (iii): Also after some algebraic manipulations, we can show that $\frac{4\overline{c}+5\widetilde{c}}{12} > \frac{3\widetilde{c}}{4}$. Step (iv): Given the result in step (ii) and (iii), we can conclude that $\alpha\left[\frac{4\overline{c}+5\widetilde{c}}{12}\right] + (1-\alpha)\left[\frac{4\widetilde{c}+5\overline{c}+3\underline{c}}{12}\right] > \frac{3\widetilde{c}}{4}$, for all $\alpha \in [0, 1]$. In particular for the case that $\alpha = \frac{18\widetilde{c}(\overline{c}-\underline{c})-(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$. Therefore, it follows that $\frac{4\overline{c}+5\widetilde{c}}{12}\left[\frac{18\widetilde{c}(\overline{c}-\underline{c})-(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}\right] + \frac{4\widetilde{c}+5\overline{c}+3\underline{c}}{12}\left[\frac{(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}\right] > \frac{3\widetilde{c}}{4}$. Therefore, $\overline{p} > \underline{p}$. □

The result in Lemma 7 is quite intuitively. In state $X = 2$, firms are symmetric. Hence, they fiercely compete for the public good provision. As consequence of such intense competition, firms bid will be very low. Therefore, the local authority's expected transfer to the winner will also be low.

Yet in the $X = 1$, the incumbent public firm faces a private firm with high expected second-period cost (private firm without outside learning). Hence, as shown in Lemma 1, the public firm does not need to be very aggressive to win the competition. If the public firm, who has higher probability of winning, bid relatively high, then local authority's expected transfer will also be relatively high.

■ **State $X = 3$.** At the contingencies which $X = 3$, the private firm expects to have lower second-period cost, whereas the public firm does not. State $X = 3$ is symmetric equivalent to the state $X = 1$, where the only difference is that the public firm in $X = 3$ behaviors as the private firm in $X = 1$, and vice-versa. Therefore, the second-period expected transfer in the state $X = 3$ is the following:

$$E[p_2|X = 3] = \overline{p}. \tag{46}$$

Note that in this state, the public authority also expects to pay high transfer to the public good provider because the private firm, who has higher probability of winning, bids relatively high.

Having computed the second-period expected transfer in all possible states in the second-period, let us characterize the ex-ante second-period expected transfer, $E[p_2]$, which is the first-period expectation of p_2 over all states described before.

With probability τ_1^F, described in Proposition 1, the private firm wins the first-period competition. As a result, only the private firm expects to have lower expected second-period cost, whereas the public one does not because it is not incumbent. Certainly, the second-period state will be $X = 3$. Therefore, as described in the previous section, the second-period expected transfer will be $\overline{p}$.

Yet when the public firm wins the second-period, which happens with probability τ_1^G, also described in Proposition 1, two possible states are possible in the second period: $X = 1$ or $X = 2$. With probability $1 - \theta$, the private firm has not outside learning. Consequently, the second period state will be $X = 1$, when only the public firm has lower expected second-period cost. In this state, the second-period expected transfer will be $\overline{p}$, as described the previous section. With probability θ the private firm has outside learning. Therefore, the second period state will be $X = 2$, when both firms expect to have lower second-period cost. Hence, the second-period expected transfer will be $\underline{p}$ as described the section above.

Hence, the ex-ante second-period expected transfer is the following:

$$E[p_2] = \tau_1^F \overline{p} + \tau_1^G [(1 - \theta)\overline{p} + \theta \underline{p}],$$

Proof of Proposition 3

From equation (32), we know that

$$E[p_2] = \tau_1^F \overline{p} + \tau_1^G [(1 - \theta)\overline{p} + \theta \underline{p}].$$

Deriving this equation with respect to θ, we obtain that

$$\frac{\partial E[p_2]}{\partial \theta} = (\overline{p} - \underline{p})\Big[\theta \frac{\partial \tau_1^F}{\partial \theta} - \tau_1^G\Big]. \tag{47}$$

In the case that $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} < \frac{8}{27}$, then from Proposition 1 we know that there exists $\widehat{\theta}$ such that for $\theta \geq \widehat{\theta}$, $\tau_1^F = 1$ and $\tau_1^G = 1$; and for $\theta < \widehat{\theta}$, $\tau_1^F = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$ and $\tau_1^G = \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$. Hence, for $\theta \geq \widehat{\theta}$, we have that $\frac{\partial E[p_2]}{\partial \theta} = 0$.

In the case that $\theta \geq \widehat{\theta}$, we have that $\frac{\partial E[p_2]}{\partial \theta} = 0$ because $\frac{\partial \tau_1^F}{\partial \theta}$ and $\frac{\partial \tau_1^G}{\partial \theta}$ are equal to zero.

Yet in the case that $\theta < \widehat{\theta}$, we have that

$$\frac{\partial E[p_2]}{\partial \theta} = \frac{(\overline{p} - \underline{p})}{2(\overline{c} - \underline{c})^2}\Big[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]\Big[\overline{c} - \underline{c} - 2\theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big].$$

From the Proposition 1, we know that $\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C) > 0$ for $\theta < \widehat{\theta}$. Hence, the sign of $\frac{\partial E[p_2]}{\partial \theta}$ will be determined by $\Big[\overline{c} - \underline{c} - 2\theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$.

Define $\psi(\theta) := \Big[\overline{c} - \underline{c} - 2\theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$. Note that $\psi(\theta)$ is a decreasing function of θ. In particular, there exists $\widetilde{\theta} = \frac{\overline{c}-\underline{c}}{2(\overline{\Pi}+\underline{\Pi}-2\Pi^C)}$ such that $\psi(\theta) < 0$ when $\theta < \widetilde{\theta}$, and $\psi(\theta) \geq 0$ when $\theta \geq \widetilde{\theta}$.

Therefore, $\frac{\partial E[p_2]}{\partial \theta} < 0$ when $\theta < \widetilde{\theta}$, and $\frac{\partial E[p_2]}{\partial \theta} \geq 0$ when $\theta \geq \widetilde{\theta}$. To conclude, note that $\widetilde{\theta} < \widehat{\theta}$. Since $\widehat{\theta} < 1$, then $\widetilde{\theta} < 1$.

In the case that $\frac{8}{27} < \frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} \leq \frac{8}{9}$, then $\tau_1^F = 1 - \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$ and $\tau_1^G = \frac{1}{2(\overline{c}-\underline{c})^2}\Big[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$. Therefore,

$$\frac{\partial E[p_2]}{\partial \theta} = \frac{(\overline{p} - \underline{p})}{2(\overline{c} - \underline{c})^2}\Big[\overline{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]\Big[\overline{c} - \underline{c} - 2\theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big].$$

As above, the sign of $\frac{\partial E[p_2]}{\partial \theta}$ will be determined by $\Big[\overline{c} - \underline{c} - 2\theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$. Hence, there exists $\widetilde{\theta} = \frac{\overline{c}-\underline{c}}{2(\overline{\Pi}+\underline{\Pi}-2\Pi^C)}$ such that $\frac{\partial E[p_2]}{\partial \theta} < 0$ when $\theta < \widetilde{\theta}$, and $\frac{\partial E[p_2]}{\partial \theta} \geq 0$ when $\theta \geq \widetilde{\theta}$.

Replacing $\overline{\Pi} + \underline{\Pi} - 2\Pi^C = \frac{8\underline{c}^2}{18(\overline{c}-\underline{c})}$ in $\widetilde{\theta}$, we find that $\widetilde{\theta}$ belongs to $(0,1)$ if and only if $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} \leq \frac{8}{9}$, what holds for this case.

Yet the case which $\frac{(\overline{c}-\underline{c})^2}{\underline{c}^2} > \frac{8}{9}$, is exactly similar to the case above. However, $\widetilde{\theta} > 1$. Therefore, for all $\theta \in (0,1)$ we have that $\frac{\partial E[p_2]}{\partial \theta} < 0$. □

Proof of Proposition 4

The total ex-ante consumers welfare was defined in (1). Replacing (31) and (32), we obtain the following expression for the total ex-ante consumers welfare:

$$W = 2u - \tau_1^G E[s_1^{*G}] - \tau_1^F E[s_1^{*F}] - \tau_1^F \overline{p} - \tau_1^G[(1-\theta)\overline{p} + \theta \underline{p}]$$

$\frac{\partial W}{\partial \theta} = -\frac{\partial E[p_1]}{\partial \theta} - \frac{\partial E[p_2]}{\partial \theta} = -\frac{\partial T(\theta)}{\partial \theta}$ with $T(\theta) = E[p_1] + E[p_2]$ and then $\frac{\partial T(\theta)}{\partial \theta} = \frac{\partial E[p_1]}{\partial \theta} + \frac{\partial E[p_2]}{\partial \theta}$.

Note that $\frac{(\bar{c}-\underline{c})^2}{\underline{c}^2} > \frac{4}{9}$ implies that $\frac{(\bar{c}-\underline{c}^2}{\underline{c})^2} > \frac{8}{27}$. Hence, we are in the case that $\tau_1^F = 1 - \frac{1}{2(\bar{c}-\underline{c})^2}\Big[\bar{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$ and $\tau_1^G = \frac{1}{2(\bar{c}-\underline{c})^2}\Big[\bar{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$. In that case, we have that

$$\begin{aligned}
\frac{\partial T(\theta)}{\partial \theta} &= \frac{\partial E[p_1]}{\partial \theta} + \frac{\partial E[p_2]}{\partial \theta} \\
&= -\frac{2(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)}{3(\bar{c} - \underline{c})^2}\Big[\bar{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]\frac{\theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)}{3} \\
&\quad + \frac{1}{2(\bar{c} - \underline{c})^2}\Big[\bar{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]^2 \frac{2\overline{\Pi} - \underline{\Pi} - \Pi^C}{3} \\
&\quad + \Big[1 - \frac{1}{2(\bar{c} - \underline{c})^2}\Big[\bar{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]^2\Big]\frac{2\overline{\Pi} + \Pi^C - 2\underline{\Pi}}{3} \\
&\quad + \frac{(\bar{p} - \underline{p})}{2(\bar{c} - \underline{c})^2}\Big[\bar{c} - \underline{c} - \frac{2\theta}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]\Big[\bar{c} - \underline{c} - 2\theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big].
\end{aligned}$$

Let us know compute $\frac{\partial T(\theta)}{\partial \theta}$ when $\theta = 0$. Doing so, we find that

$$\frac{\partial T(\theta)}{\partial \theta}|_{\theta=0} = \frac{\overline{\Pi} - \underline{\Pi} - (\bar{p} - \underline{p})}{2} \tag{48}$$

From the results before, we can find that $\overline{\Pi} - \underline{\Pi} = \frac{2\underline{c}}{3}$. From Lemma 7, we know that $\frac{4\tilde{c}+5\tilde{c}}{12} < \bar{p} < \frac{4\tilde{c}+5\bar{c}+3\underline{c}}{12}$. Hence, $\frac{4\tilde{c}+5\tilde{c}}{12} - \underline{p} > \bar{p} - \underline{p}$. Therefore, it is enough to show that $\frac{2\underline{c}}{3} \geq \frac{4\tilde{c}+5\tilde{c}}{12} - \underline{p}$ to show that $\frac{\partial T(\theta)}{\partial \theta}|_{\theta=0}$ is positive. Computing $\frac{4\tilde{c}+5\tilde{c}}{12} - \underline{p}$, we find that it is equal to $\frac{2\underline{c}}{3}$, then $\frac{\partial T(\theta)}{\partial \theta}|_{\theta=0}$ is positive.

We will show that $\frac{\partial T(\theta)}{\partial \theta}$ is strictly increasing in θ. Firstly, note that

$$\frac{\partial^2 E[p_1]}{\partial \theta^2} = -\frac{4}{9(\bar{c} - \underline{c})^2}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)^2\Big[\bar{c} - \underline{c} - \theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$$

and

$$\frac{\partial^2 E[p_2]}{\partial \theta^2} = \frac{4(\bar{p} - \underline{p})}{3(\bar{c} - \underline{c})^2}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)^2\Big[\bar{c} - \underline{c} - \theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]$$

Therefore,

$$\frac{\partial^2 T(\theta)}{\partial \theta^2} = \frac{4}{3}\Big[\bar{c} - \underline{c} - \theta(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big]\frac{(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)^2}{(\bar{c} - \underline{c})^2}\Big[\bar{p} - \underline{p} - \frac{1}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)\Big] \tag{49}$$

Note that it is positive if $\bar{p} - \underline{p} - \frac{\overline{\Pi}+\underline{\Pi}-2\Pi^C}{3}$. Using the same argument above, $\bar{p} - \underline{p} > \frac{4\tilde{c}+5\bar{c}+3\underline{c}}{12} - \underline{p}$, since $\bar{p} > \frac{4\tilde{c}+5\bar{c}+3\underline{c}}{12}$. Therefore, it is enough to show that $\frac{4\tilde{c}+5\bar{c}+3\underline{c}}{12} - \underline{p} > \frac{\overline{\Pi}+\underline{\Pi}-2\Pi^C}{3}$ to show that $\frac{\partial^2 T(\theta)}{\partial \theta^2}$ is positive. Note that $\frac{4\tilde{c}+5\bar{c}+3\underline{c}}{12} - \underline{p} = \frac{\underline{c}}{3}$. After some algebraic manipulations, we can show that $\frac{4\tilde{c}+5\bar{c}+3\underline{c}}{12} - \underline{p} > \frac{1}{3}(\overline{\Pi} + \underline{\Pi} - 2\Pi^C)$ if $\frac{\underline{c}}{54(\bar{c}-\underline{c})}(18\bar{c} - 26\underline{c}) \geq 0$. It holds when $c \geq \frac{13}{9}$. Note that the last one holds when $\frac{(\bar{c}-\underline{c})^2}{\underline{c}^2} > \frac{4}{9}$.

Hence, if $\frac{\partial^2 T(\theta)}{\partial \theta^2} > 0$ and $\frac{\partial T(\theta)}{\partial \theta}|_{\theta=0} > 0$, then $\frac{\partial T(\theta)}{\partial \theta} > 0$ for all θ. Therefore, $\frac{\partial W}{\partial \theta} < 0$ for all θ. $\square$

Proof of Implication 1

Before showing the proofs, let us define $\overline{\tau} = \frac{18\widetilde{c}(\overline{c}-\underline{c})-(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$, $\underline{\tau} = \frac{(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$, and $\tau^C = \frac{1}{2}$. Note that, form Lemma 1 we have that $\overline{\tau} > \tau^C > \underline{\tau}$.

Let us first consider the case that the public firm is incumbent. In this case, the probability that public firm wins is $\theta\tau^C+(1-\theta)\underline{\tau}$. Yet the probability that the private firm wins is $\theta\tau^C+(1-\theta)\underline{\tau}$. We want to show that $\theta\tau^C+(1-\theta)\underline{\tau} > \theta\tau^C+(1-\theta)\underline{\tau}$. This inequality is equivalent to $\theta(\overline{\tau}-\underline{\tau})+(\tau^C-\underline{\tau}) > 0$, which always hold.

Let us now consider the case that the private firm is incumbent. In this case, the probability that public firm wins is $\underline{\tau}$. Yet the probability that the private firm wins is $\overline{\tau}$. Clearly, the probability that public firm wins is lower than the probability that the private firm wins since the following inequality holds $\overline{\tau} > \tau^C > \underline{\tau}$. □

Proof of Implication 3

We want to show that $E[p_2] > E[p_1]$. We know that $E[p_1] = \tau_1^G E[s_1^{*G}] + \tau_1^F E[s_1^{*F}]$, and $E[p_2] = \tau_1^F \overline{p} + \tau_1^G[(1-\theta)\overline{p} + \theta\underline{p}]$.

First note that $E[p_1] < \tau_1^F[E[s_1^{*G}] + E[s_1^{*F}]]$, since $\tau_1^F > \tau_1^G$. Therefore, it is sufficient to show that $E[p_2] > \tau_1^F[E[s_1^{*G}]+E[s_1^{*F}]]$. However, before it, let us compute $\tau_1^F[E[s_1^{*G}]+E[s_1^{*F}]]$. Replacing the values for $E[s_1^{*G}]$ and $E[s_1^{*F}]$ in that equation, we obtain that

$$\tau_1^F[E[s_1^{*G}] + E[s_1^{*F}]] = \tau_1^F\left(\frac{3\overline{c}}{2} + \frac{\underline{c}}{2} - \frac{2}{9}\underline{c}(2-\theta)\right)$$

Note that this expression is lower than $\tau_1^F\left(\frac{3\overline{c}}{2} + \frac{5\underline{c}}{18}\right)$. Therefore, it is sufficient to show that $E[p_2] > \tau_1^F\left(\frac{3\overline{c}}{2} + \frac{5\underline{c}}{18}\right)$.

Yet $E[p_2] > \tau_1^G[\overline{p}+\underline{p}]$. Hence, it is sufficient to show that $\tau_1^G[\overline{p}+\underline{p}] > \tau_1^G[\overline{p}+\underline{p}]$. Replacing $\overline{p}$ and $\underline{p}$, which are given by the following expressions:

$$\overline{p} = \frac{4\overline{c}+5\widetilde{c}}{12}\left[\frac{18\widetilde{c}(\overline{c}-\underline{c})-(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}\right] + \frac{4\widetilde{c}+5\overline{c}+3\underline{c}}{12}\left[\frac{(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}\right]$$

and

$$\underline{p} = \frac{3\widetilde{c}}{4}$$

We find that $\tau_1^G[\overline{p}+\underline{p}] > \tau_1^G[\overline{p}+\underline{p}]$ holds. □

Proof of Implication 2

Before providing the proof of Implication 2, let us define $p_1 = \frac{4\overline{c}+5\widetilde{c}}{12}$ and $p_2 = \frac{4\widetilde{c}+5\overline{c}+3\underline{c}}{12}$. From Lemma 7, we know that $p_2 > \overline{p} > p_1$.

The expected second-period transfer under private provision is equal to

$$E[p_2|\text{if private ownership at t=1}] = \tau_1^F \overline{\tau} p_1 + \tau_1^G [\theta \tau^C \underline{p} + (1-\theta) \underline{\tau} p_2]$$

and the expected second-period transfer under public provision is equal to

$$E[p_2|\text{if public ownership at t=1}] = \tau_1^F \underline{\tau} p_2 + \tau_1^G [\theta \tau^C \underline{p} + (1-\theta) \overline{\tau} p_1]$$

where $\overline{\tau} = \frac{18\widetilde{c}(\overline{c}-\underline{c})-(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$, $\underline{\tau} = \frac{(2\widetilde{c}+3\underline{c}+\overline{c})^2}{18\widetilde{c}(\overline{c}-\underline{c})}$, and $\tau^C = \frac{1}{2}$. Note that, form Lemma 1 we have that $\overline{\tau} > \tau^C > \underline{\tau}$.

Given that, we can compute $E[p_2|\text{if private ownership at t=1}] - E[p_2|\text{if public ownership at t=1}]$. Taking the expressions above, we find that $E[p_2|\text{if private ownership at t=1}]$ - $E[p_2|\text{if public ownership at t=1}] = (p_1\overline{\tau} - p_2\underline{\tau})(\tau_1^F - \tau_1^G(1-\theta))$. Note that $(\tau_1^F - \tau_1^G(1-\theta))$ is always positive since $\tau_1^F > \tau_1^G$, and $\theta \in (0,1)$. Therefore, the sign of $E[p_2|\text{if private ownership at t=1}] - E[p_2|\text{if public ownership at t=1}]$ is determined by $(p_1\overline{\tau} - p_2\underline{\tau})$. Replacing the values p_1, $\overline{\tau}$, p_2 and $\underline{\tau}$, defined in the beginning of the proof, in that expression, we obtain that $(p_1\overline{\tau} - p_2\underline{\tau})$ is equal to

$$\frac{4\overline{c} + 5\widetilde{c}}{12} - \underline{\tau}\frac{\overline{c} + 3\underline{c}}{12}.$$

Because that $\underline{\tau} \in (0, \frac{1}{2})$, then the expression above is greater than

$$\frac{4\overline{c} + 5\widetilde{c}}{12} - \frac{\overline{c} + 3\underline{c}}{12}.$$

This expression is equal to $\frac{8(\overline{c}-\underline{c})}{12}$, which is positive. Hence, $E[p_2|\text{if private ownership at t=1}] - E[p_2|\text{if public ownership at t=1}]$. □