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Small Open Economy**
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The Natural Rate of Interest in a Small Open Economy

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The goal of this paper is two-fold. Firstly, this paper shows that the natural rate of interest in a small open economy, with access to the world capital markets, is equal to the international real rate of interest. We show this property by using the infinitely-lived overlapping generations model and we use this model to analyze both fixed and flexible exchange rate regimes. Secondly, this paper also shows that the empirical implausibility hypothesis embedded in the infinitely-lived representative-agent model, with complete asset markets, turns this framework not appropriate for a small open economy.

1. Introduction

Obstfeld and Rogoff (1995) initiated a research program that has become known as "The New Open Economy Macroeconomics" (NOEM). This research is based on dynamic intertemporal models featuring rational expectations, imperfect competition and nominal-price rigidity.[1] One goal of this research is to build a model to replace or to update the workhorse small open economy model of Mundell-Fleming- Dornbusch, which has resisted the micro-foundations revolution, in spite of the well-known Lucas critique. Recent work by Galí and Monacelli (2005), henceforth GM, present a model for a small open economy that does not influence foreign output, foreign price level and foreign interest rate, which attempts to fill this gap in the literature.[2]

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[1] Lane (2001) provides a comprehensive survey of the early new open economy macroeconomics literature.

[2] For other contributions to the literature on monetary policy in small open economies see, for example, Clarida, Galí and Gertler (2001), McCallum and Nelson (2000), Svensson (2000).

GM's model uses the infinitely-lived intertemporal optimizing representative-agent model. It is common knowledge [see Barro and Sala-i-Martin (1995), Chapter 3] that this model, for a small open economy with perfect access to the world capital market and a fixed rate of time preference, yields awkward consequences that are not tenable either from a theoretical or from an empirical point of view. If the two rates were different, there would be opportunities for arbitrage and capital flows that lead to paradoxical conclusions, either the country owns the world's wealth or it mortgages all of its capital and all of its wage income. These results are incompatible with a steady state. In a closed economy, the real rate of interest is equal to the rate of time preference in steady state. If they are different, the stock of capital adjusts to make up the difference. In an open economy, the real rate of interest is equal to the international real rate of interest in steady state. In this case there is no mechanism to make the adjustment between the international real rate and the rate of time preference. The difference between the two rates, whether positive or negative, would imply consequences that are counterfactuals.[3]

Four modifications of the standard infinitely-lived intertemporal optimizing representative-agent model have been used in the literature to solve this problem [see Schimitt-Grohé and Uribe (2003)]. They are: i) endogenous time preference à la Uzawa (1968); ii) endogenous risk premium; iii) convex portfolio adjustment costs; and iv) complete asset markets. The endogenous time preference has to be such that agents become more impatient when consumption increases, which is a very counterintuitive hypothesis. The interest rate premium and convex portfolio adjustment costs hypotheses are ad hoc and, as such, can be discarded because it lacks microeconomic foundations. The added hypothesis of complete asset markets to the standard representative-agent model is indeed a very strong hypothesis and has been rejected by empirical evidence [see, for example, Obstfeld and Rogoff (2001), and references therein]. Thus, the four modifications are not particularly compelling for small open economies with perfect access to the world capital market.

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[3] Obstfeld and Rogoff (1995) set up a two-country model, where home and foreign individuals have identical preferences and the two countries have the same technology. This setup avoids the problem of natural rate inconsistency because the world is a closed economy. Thus, in such a case the real rate of interest is equal to the rate of time preference in steady state.

GM uses the complete asset market hypothesis and they did not stress the fact that in their small open economy, with perfect capital mobility, the natural rate of interest would not be equal to the international real rate of interest. The natural rate of interest in their paper is equal to the international rate of interest plus a constant rate of change of the real exchange rate. Underlying this natural rate there is a highly stringent condition not observable in the real world, namely that the country currency becomes weaker if the country grows at a rate greater than the world economy. Furthermore, GM's model implicit assumes perfect synchronization between domestic and international business cycles. Otherwise, the world economy has to follow an unstable Taylor rule.

The natural rate of interest is a benchmark to measure the stance of monetary policy. Furthermore, to use monetary policy rules, such as Taylor(1993), an estimate of the natural rate is required. The literature that deals with the measurement of the natural rate [see, for example, Laubach and Williams (2003), Giammarioli and Valla (2004)] has used closed economy models, since big open economies are treated as closed ones, and has not addressed the open economy case. This paper fills this gap in the literature and provides a framework to construct natural rate series for the small open economy based on economic theory. This is accomplished by using an overlapping generations model that yields a unique steady state when the rate of time preference is different from the international real rate of interest.

The goal of this paper is two-fold. Firstly, this paper shows that the natural rate of interest in a small open economy is equal to the international real rate of interest. We show this property by using the infinitely-lived overlapping generations model [Weil (1989)] and we use this model to analyze both fixed and flexible exchange rate regimes. Secondly, this paper also shows that the empirical implausibility hypothesis embedded in GM's model, obtained from the infinitely-lived representative-agent model, turns this framework not appropriate as a Mundell-Fleming-Dornbusch replacement.

This paper is organized as follows. Section 2 shows that GM's representative agent small open economy model is valid under two stringent conditions, one on the behavior of the real exchange rate and another one related to the real rate of interest monetary rule, which are not supported on empirical grounds. Section 3 derives the IS curve from the infinitely-lived overlapping generations model, in which the natural rate of interest is equal to the international real rate of interest. In this section we use

McCallum and Nelson (2000) framework that avoids the cumbersome algebra usually found in NOEM models, which allows a very simple and straightforward derivation of the IS curve, without loss of generality. We also use deterministic variables because a stochastic environment is not necessary to show the natural rate consistency embedded in the overlapping model. Section 4 applies this model to a fixed exchange rate system and Section 5 deals with the flexible exchange rate system. The models in these two sections cannot be reduced to a two equation dynamic system, in the rate of inflation and output gap, like the closed economy models, because the behavior of the current account affects net foreign asset and through the goods market output gap and, hence, inflation. Section 6 concludes.

2. GM's Representative Agent Open Economy Model

GM's framework can be reduced to a log-linearized canonical two-equation dynamical system, consisting of a new Keynesian Phillips curve for domestic inflation (π_t) and an IS-type curve for the output gap (x):

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa_\alpha x_t \tag{1}$$

$$x_t = E_t \{x_{t+1}\} - \frac{1}{\sigma_\alpha} (i_t - E_t \{\pi_{t+1}\} - \bar{r}_t) \tag{2}$$

We use the notation: $\rho = \beta^{-1} - 1$ is the rate of time preference, i_t is the nominal rate of interest, $\kappa_\alpha = \lambda(\sigma_\alpha + \varphi)$, $\lambda = [(1 - \beta\theta)(1-\theta)]/\theta$, $(1-\theta)^{-1}$ can be interpreted as the average period between price adjustments in the Calvo-type staggered price-setting, φ^{-1} is the labor supply elasticity, $\sigma_\alpha = 1/\sigma [(1-\alpha) + \alpha\omega]$, $\omega = [\gamma + (1-\alpha)(\eta-\sigma)]/\sigma$, σ is the elasticity of intertemporal substitution of consumption, α is an openness index and can be interpreted as the share of domestic consumption allocated to imported goods, γ is the elasticity of substitution between goods produced in different foreign countries, η is the elasticity of substitution between domestic and foreign goods. The natural rate of interest is given by:

$$\bar{r}_t = \rho + \frac{1}{\sigma_a} E_t \, \Delta \, (\bar{y}_{t+1} - y^*_{t+1}) + \frac{1}{\sigma} E_t \, \Delta \, y^*_{t+1} \qquad (3)$$

Goods market clearing, world market clearing and the assumption of complete markets leads to a relationship linking domestic output, world output and the terms of trade:

$$y_t = y^*_t + \frac{1}{\sigma_a} s_t \qquad (4)$$

This equation implies that:

$$\bar{y}_{t+1} = y^*_{t+1} + \frac{1}{\sigma_a} \bar{s}_{t+1} \qquad (5)$$

Plugging this equation into the definition of the natural rate of interest we obtain:

$$\bar{r}_t = \rho + \frac{1}{\sigma} E_t \, \Delta \, y^*_{t+1} + E_t \, \Delta \, \bar{s}_{t+1}$$

$$= \rho + \frac{1}{\sigma} E_t \, \Delta \, \bar{y}^*_{t+1} + E_t \, \Delta \, \bar{s}_{t+1} + \frac{1}{\sigma} E_t \, \Delta \, (y^*_{t+1} - \bar{y}^*_{t+1}) \qquad (6)$$

GM's framework assumes complete financial markets. In this environment a version of the uncovered interest parity, which abstracts from a risk premium term, is obtained:

$$i_t - i^*_t = E_t \left\{ \Delta \, e_{t+1} \right\} \qquad (7)$$

where e_t is the nominal exchange rate. The uncovered interest parity for the real interest rate differential can be written as:

$$i_t - E_t \left\{ \pi_{t+1} \right\} = i^*_t - E_t \left\{ \pi^*_{t+1} \right\} + E_t \left\{ \Delta \, s_{t+1} \right\} \qquad (8)$$

where s is the terms of trade. Plugging this result into the output gap equation we obtain the following expression:

$$x_t = E_t\left\{x_{t+1}\right\} - \frac{1}{\sigma_\alpha}\left(r_t^* + E_t\left\{\Delta s_{t+1}\right\} - \bar{r}_t\right) \tag{9}$$

where $r_t^* = i_t^* - E_t\left\{\pi_{t+1}^*\right\}$ is the foreign real rate of interest. In the perfect foresight steady state the natural rate of interest is equal to the foreign real interest rate plus the long run change in the terms of trade, since $\Delta s \neq 0$. We turn now to a discussion of this statement.[4]

It follows from (5) that the long run change in the terms of trade is proportional to the difference between the rate of change of domestic potential output and the rate of change o world output:

$$E_t \Delta \bar{s}_{t+1} = \sigma_a E_t \Delta (\bar{y}_{t+1} - y_{t+1}^*) \tag{10}$$

.

This proposition says that if a country grows at a rate greater than the rate of growth of the world than its real exchange rate keeps depreciating all the time. Unfortunately, the empirical evidence does not support this hypothesis. Hence, the representative agent small open economy rests on unsafe empirical grounds. We may conclude that the natural rate of interest in a small open economy, in a representative agent model, is equal to the international rate of interest just when both rates of output growth, domestic and world, are equal. Otherwise, they are different.

The world natural rate of interest is defined by:

$$\bar{r}_t^* = \rho + \frac{1}{\sigma} E_t \Delta \bar{y}_{t+1}^* \tag{11}$$

Combining (11) with the natural rate of interest equation (6) we conclude that domestic natural rate of interest has three components: i) world natural rate of interest; ii) expected rate of change of the terms of trade, and iii) a term proportional to the gap between the rate of change of world output and world potential output. Hence,

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[4] In their calibrated version of the Canadian economy the natural rate of interest in the Taylor rule used by GM is equal to the time discount rate [GM, p.723].

$$\bar{r}_t = \bar{r}_t^* + E_t \ \Delta \ \bar{s}_{t+1} + \frac{1}{\sigma} E_t \ \Delta \ (y_{t+1}^* - \bar{y}_{t+1}^*) \qquad (12)$$

When the small open economy is in equilibrium the domestic natural rate of interest is equal to the international rate of interest plus the expected rate of change of the terms of trade:

$$r_t^* + E_t \ \Delta \ \bar{s}_{t+1} = \bar{r}_t \qquad (13)$$

Combining (12) and (13) we conclude that the world rate of interest, according to this model, has to follow the Taylor rule:

$$r_t^* = \bar{r}_t^* + \frac{1}{\sigma} E_t \ \Delta \ (y_{t+1}^* - \bar{y}_{t+1}^*)$$

There is no empirical evidence that supports this hypothesis. We may conclude that the assumption of complete asset markets is not able to save the small economy representative agent model.

3. Overlapping Generations Open Economy Model

An agent born at time s lives forever and maximizes at time t her utility function,

$$\int_t^\infty e^{-\rho \, (v-t)} u \left[C(s,v) \right] \, dv$$

subject to the flow budget constraint:

$$\dot{A}(s,v) = y(v) - C(s,v) - g(v) + r \, A(s,v)$$

where ρ is the rate of time preference, $u[C(s, v)]$ is the instantaneous utility function, $C(s, v)$ stands for the consumption at time v by an agent born at time s, $A(s, v)$ are real financial assets, $y(v)$ is her income and $g(v)$ is government expenditure supposed to be

financed by lump sum taxes. The income and the lump sum tax are not cohort specific. In this model asset markets are incomplete, agents have access to a foreign bond, r is the domestic real rate of interest, UIP condition holds and the international rate of interest is exogenous to this small open economy. Newly born agents are not linked altruistically to existing cohorts and they are born with only non financial wealth but no financial assets.[5] Thus: $A(v, v) = 0$. The first order condition of this problem yields the Euler equation:

$$\frac{\dot{C}(s,v)}{C(s,v)} = \sigma(r - \rho)$$

The aggregate Euler equation contains an adjustment term for consumption by the newborn generation ($C(t,t)$):[6]

$$\frac{\dot{C}}{C} = \sigma(r - \rho) + n\left[C(t,t) - C(t)\right]$$

Since consumption is proportional to total wealth, $C(t) = \phi\left[A(t) + NF(t)\right]$, where $\phi = r + \sigma(\rho - r) > 0$, A denotes the economy's total net foreign assets, NF stands for non-financial wealth and $C(t,t) = \phi\left[A(t,t) + NF(t)\right] = \phi NF(t)$ because the agent is born without financial assets. Thus, we have: $C(t,t) - C(t) = -\phi A(t)$. Making use of this result the aggregate Euler equation is given by:

$$\frac{\dot{C}}{C} = \sigma(r - \rho) - n\,\phi\,\frac{A}{C}$$

This small open economy with perfect access to the world capital market can have a steady state for aggregate consumption ($\dot{C} = 0$) even when the rate of time preference is different from the real interest rate. This small open economy is a creditor country when the real interest rate is greater than the rate of time preference and a debtor country when the rate of time preference is greater than the real interest rate. By adding and subtracting $n\theta\overline{A}/\overline{C}$ to this expression we get:

$$\dot{c} = \sigma(r - \overline{r}) - n\theta\left(\frac{A}{C} - \frac{\overline{A}}{\overline{C}}\right)$$

The natural rate of interest is given by:

$$\overline{r} = \rho + \frac{n\theta}{\sigma}\frac{\overline{a}}{\overline{c}_y}$$

In this setup the adjustment between the rate of time preference and the international real rate of interest is through the asset consumption ratio, which can be positive or negative depending on the degree of impatience of the small open economy compared to the world real rate of interest. We use the following notation $\overline{a} = \overline{A}/\overline{y}$ and $\overline{c}_y = \overline{C}/\overline{y}$, with net foreign asset and consumption normalized by potential output. The first order linear expansion of $f(A/C) = A/C$ is equal to:

$$\frac{A}{C} = \frac{\overline{A}}{\overline{C}} + \frac{1}{\overline{C}}(A - \overline{A}) - \frac{1}{\overline{C}^2}\overline{A}(C - \overline{C})$$

This expression can be rewritten as:

$$\frac{A}{C} = \frac{\overline{A}}{\overline{C}} + \frac{\overline{y}}{\overline{C}}\left(\frac{A}{\overline{y}} - \frac{\overline{A}}{\overline{y}}\right) - \frac{\overline{A}/\overline{y}}{\overline{C}/\overline{y}}\left(\frac{C - \overline{C}}{\overline{C}}\right)$$

Thus, the gap of the relation wealth/consumption depends on the wealth and consumption gaps according to:

$$\frac{A}{C} - \frac{\overline{A}}{\overline{C}} = \frac{1}{\overline{c}_y}(a - \overline{a}) - \frac{\overline{a}}{\overline{c}_y} \log \frac{C}{\overline{C}}$$

By substituting this expression in the equation for the rate of growth of consumption ($\dot{c}$), we obtain

$$\dot{c} = \sigma(r - \overline{r}) - \frac{n\theta}{\overline{c}_y}(a - \overline{a}) - \frac{n\theta\overline{a}}{\overline{c}_y}(c - \overline{c})$$

The small letter c denotes the logarithm of consumption: $c = \log C$.

The IS curve for a small open economy can be obtained from an infinitely-lived overlapping generations model by combining the aggregate consumption Euler equation with McCallum and Nelson (2000) approach that assumes that all imports and labor are used as inputs in the production of domestic goods. Thus, output in this small open economy can be written as: $y = \omega_1 c + \omega_2 g + \omega_3 ex$, where c, y, g, ex are logarithms of consumption, real output, government consumption and exports, while ω_i is the steady-state ratio of the corresponding variable. Output gap is the difference between real output and potential output (assumed constant, for simplicity).

$$x = y - \overline{y} = \omega_i(c - \overline{c}) + \omega_2(g - \overline{g}) + \omega_3(ex - \overline{e}x)$$

By taking time derivative of the output gap we obtain:

$$\dot{x} = \omega_i\dot{c} + \omega_2\dot{g} + \omega_3\dot{e}x$$

The exports equation is specified by:

$$ex = y^* + \xi s + \kappa$$

where y^* is world output, ξ is the elasticity of substitution between imported materials and labor in production, κ is a constant and s is the terms of trade.

By combining the equations of $\dot{c}, x \text{ and } \dot{x}$ and the exports equation we get the IS curve:

$$\dot{x} = \beta_x x + \beta_r(r - \overline{r}) - \beta_a(a - \overline{a}) - \beta_q(q - \overline{q})$$

This equation states that the output gap rate of change depends on the output gap level and the gaps of the interest rate, wealth and real exchange rate. Thus, net foreign assets plays an explicit role in this IS curve. The parameters of this curve depend on structural parameters according to:

$$\beta_x = \frac{n\theta\overline{a}}{\overline{c}_y} \quad ; \beta_r = \omega_1\sigma$$

$$\beta_a = \frac{\omega_1 n\theta}{\overline{c}_y} \quad ; \beta_q = \frac{n\theta\overline{a}\omega_3\xi}{(1-\alpha)\overline{c}_y}$$

The coefficients β_r e β_a are positive. On the other hand the coefficients β_x e β_q can be either positive or negative. They are positive if the country is a net creditor $(\overline{a} > 0)$ and negative if the country is a net debtor $(\overline{a} < 0)$.

The accumulation of net foreign assets in a small open economy overlapping generations model is given by the balance of payments current account:

$$\dot{A} = (r - n)A + Ex$$

This equation, when the variables are normalized by the potential output of the economy, can be written as:

$$\dot{a} = (r - n)a + \frac{Ex}{\overline{y}}$$

In steady state $(\dot{a} = 0)$ we get:

$$0 = (\overline{r} - n)\overline{a} + \frac{\overline{Ex}}{\overline{y}}$$

Hence,

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$$\dot{a} = (r - n)a - (\bar{r} - n)\bar{a} + \frac{Ex - \overline{Ex}}{\bar{y}}$$

The first order expansion of both interest income ($r\,a$) and net exports equations are, respectively:

$$r\,a = \bar{r}\,\bar{a} + \bar{r}(a - \bar{a}) + \bar{a}(r - \bar{r})$$

$$\frac{Ex}{\bar{y}} = \frac{\overline{Ex}}{\bar{y}} + \frac{\bar{S}\;Ex'}{\bar{y}}\left(\frac{S - \bar{S}}{\bar{S}}\right) = \frac{\overline{Ex}}{\bar{y}} + \varepsilon(s - \bar{s})$$

Substituting these expressions in the equation of $\dot{a}$ yields:

$$\dot{a} = \alpha_a(a - \bar{a}) + \alpha_r(r - \bar{r}) + \alpha_q(q - \bar{q})$$

The parameters of this equation depend on the structural parameters of the model according to:

$$\alpha_a = \bar{r} - n \qquad \alpha_r = \bar{a}, \quad \alpha_q = \varepsilon/(1 - \alpha)$$

where we take into account the relationship between the terms of trade and the real exchange rate: $q = (1 - \alpha)\,s$. The coefficients α_a *and* α_q are positive. The sign of α_r depends whether the country is a creditor $(\bar{a} > 0)$ or a debtor $(\bar{a} < 0)$.

4. Fixed Exchange Rate Regime

The new keynesian model of a small open economy under a fixed exchange rate regime has an IS curve derived from the Euler equation, a Phillips curve à la Calvo, an uncovered interest parity equation, a monetary policy rule whereas the central bank pegs the exchange rate and an equation for the balance of payments current account. Calvo's Phillips curve is a continuous–time staggered prices formulation and we keep the model simple excluding the change of the terms of trade from this curve. The initial conditions

of the model suppose that the price level, the real exchange rate and the net foreign asset are predetermined variables. The other variables are free and are allowed to jump in response to new information. The model has the following specification:

IS: $\quad \dot{x} = \beta_x\, x + \beta_r\left(r - \bar{r}\right) - \beta_a\left(a - \bar{a}\right) - \beta_q\left(q - \bar{q}\right)$

PC: $\quad \dot{\pi} = -\delta\, x$

UIP: $\quad \dot{q} = r - \bar{r}$

MPR: $\quad e = ct$

BP: $\quad \dot{a} = \alpha_a\left(a - \bar{a}\right) + \alpha_r\left(r - \bar{r}\right) + \alpha_q\left(q - \bar{q}\right)$

IC: $\quad$ Given $p(0)$, $q(0)$ and $a(0)$

The terms of trade is defined by:

$$s = e + p^* - p$$

Taking time derivatives on both sides of this expression and taking into account that the nominal exchange rate is fixed $\left(\dot{e} = 0\right)$, we get:

$$\dot{s} = \dot{e} + \pi^* - \pi = \pi^* - \pi = \frac{\dot{q}}{1 - \alpha} = \frac{r - \bar{r}}{1 - \alpha}$$

Substituting the inflation gap for the real interest gap in the tree equations, namely the IS curve, the uncovered interest parity and the current account, we get, respectively:

$$\dot{x} = \beta_x x - \beta_\pi\left(\pi - \pi^*\right) - \beta_a\left(a - \bar{a}\right) - \beta_s\left(s - \bar{s}\right)$$

$$\dot{s} = -\left(\pi - \pi^*\right)$$

$$\dot{a} = -\alpha_\pi\left(\pi - \pi^*\right) + \alpha_s\left(s - \bar{s}\right) + \alpha_a\left(a - \bar{a}\right)$$

where $\beta_\pi = (1 - \alpha)\,\beta_r$ and $\alpha_\pi = (1 - \alpha)\,\alpha_r$. The dynamic system of the small open economy new Keynesian model, for a fixed exchange rate system, has four differential equations in the following variables: rate of inflation, output gap, real exchange rate,

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and net foreign wealth. The last two variables, the real exchange rate and net foreign wealth, are predetermined. The linear approximation of the dynamic system around the steady state is given by the four-differential equation in π, x, q and a:

$$\begin{bmatrix} \dot{\pi} \\ \dot{x} \\ \dot{q} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} 0 & -\delta & 0 & 0 \\ -\beta_\pi & \beta_x & -\beta_s & -\beta_a \\ -1 & 0 & 0 & 0 \\ -\alpha_\pi & 0 & \alpha_s & \alpha_a \end{bmatrix} \begin{bmatrix} \pi - \pi^* \\ x \\ q - \overline{q} \\ a - \overline{a} \end{bmatrix}$$

The Jacobian matrix of this system is given by:

$$J = \begin{bmatrix} 0 & -\delta & 0 & 0 \\ -\beta_\pi & \beta_x & -\beta_s & -\beta_a \\ -1 & 0 & 0 & 0 \\ -\alpha_\pi & 0 & \alpha_s & \alpha_a \end{bmatrix}$$

Hence, the determinant and the trace of this matrix are, respectively:

$$|J| = \delta[\alpha_s \beta_a - \alpha_a \beta_s]$$

$$tr\ J = \beta_x + \alpha_a$$

For this system to exhibit saddle path stability with two positive roots and two negative roots the determinant has to be positive. This determinant will also be positive if all four roots have the same sign (either positive or negative). Thus, we have to prove that all roots have not the same sign. For this to be the case the coefficient of μ^2 of the characteristic equation should be negative [Sydsaeter, Strom and Berck (2000), p.8]:

$$\sum_{i<j} \mu_i \mu_j = coefficient\ of\ \mu^2$$

It is easy to verify that if all characteristics roots have the same sign the coefficient of μ^2 would be positive. The characteristic equation of this dynamic system is given by:

$$\begin{vmatrix} -\mu & -\delta & 0 & 0 \\ -\beta_\pi & \beta_x - \mu & -\beta_s & -\beta_a \\ -1 & 0 & -\mu & 0 \\ -\alpha_\pi & 0 & \alpha_s & \alpha_s - \mu \end{vmatrix} = 0$$

Solving this determinant by the cofactor expansion along the first line we get the characteristic equation:

$$\mu^4 - (\alpha_a + \beta_x)^3 + (\alpha_a \beta_x - \delta\beta_\pi)\mu^2 +$$
$$\delta(\alpha_a \beta_\pi + \alpha_r \beta_a + \beta_s)\mu + \delta(\alpha_s \beta_a \alpha_a \beta_s) = 0$$

The coefficient of μ^2 should be negative for the dynamic system to have a saddle path with two positive and two negative roots. This is the case if the following inequality holds:

$$\beta_\pi > \frac{\beta_x \alpha_a}{\delta}$$

Thus, if this condition holds the system exhibits saddle path stability. Setting to zero the constants corresponding to the unstable roots, the dynamic system solution, with two negative characteristic roots, $\mu_1 > \mu_2$, is given by:

$$\pi = \pi^* + c_1 v_{11} e^{\mu_1 t} + c_2 v_{21} e^{\mu_2 t}$$

$$x = c_1 v_{12} e^{\mu_1 t} + c_2 v_{22} e^{\mu_2 t}$$

$$q = \bar{q} + c_1 v_{13} e^{\mu_1 t} + c_2 v_{23} e^{\mu_2 t}$$

$$a = \bar{a} + c_1 v_{14} e^{\mu_1 t} + c_2 v_{24} e^{\mu_2 t}$$

The constants c_1 and c_2 are obtained from the initial conditions where the elements v_{ij} of the eigenvector associated to the negative roots come from the solution of the linear system:

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$$\begin{bmatrix} -\mu & -\delta & 0 & 0 \\ -\beta_\pi & \beta_x - \mu & -\beta_s & -\beta_a \\ -1 & 0 & -\mu & 0 \\ -\alpha_\pi & 0 & \alpha_s & \alpha_a - \mu \end{bmatrix} \begin{bmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \\ v_{i4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The first and the third equations of this system yield the inequalities:

$$\frac{v_{i1}}{v_{i2}} = -\frac{\delta}{\mu} > 0$$

$$\frac{v_{i1}}{v_{i3}} = -\mu > 0$$

When the fourth equation is combined with the third we get the inequality:

$$\frac{v_{i3}}{v_{i4}} = -\frac{\alpha_a - \mu}{\alpha_\pi \mu + \alpha_s} < 0$$

These three inequalities will provide crucial information to derive the behavior of the dynamic system. Dividing the inflation gap by the real interest gap and taking its limit when time goes to infinite we obtain:

$$\lim_{t \to \infty} \frac{\pi - \pi^*}{x} = \lim_{t \to \infty} \frac{c_1 v_{11} e^{\mu_1 t} \left[1 + \frac{c_2 v_{21}}{c_1 v_{11}} e^{(\mu_2 - \mu_1)t} \right]}{c_1 v_{12} e^{\mu_1 t} \left[1 + \frac{c_2 v_{22}}{c_1 v_{12}} e^{(\mu_2 - \mu_1)t} \right]}$$

When $t \to \infty$, since $\mu_2 - \mu_1 < 0$, the terms into brackets, both in the numerator and the denominator, converges to one. Thus,

$$\lim_{t \to \infty} \frac{\pi - \pi^*}{x} = \frac{v_{11}}{v_{12}} > 0$$

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This equation indicates that the inflation gap ant the output gap, when the system converges asymptotically to the dominant ray [Calvo(1987)], move in the same direction. We may use the same procedure to prove that:

$$\lim_{t \to \infty} \frac{\pi - \pi^*}{q - \bar{q}} = \frac{v_{11}}{v_{13}} > 0$$

$$\lim_{t \to \infty} \frac{a - \bar{a}}{q - \bar{q}} = \frac{v_{14}}{v_{13}} < 0$$

The first inequality shows that the inflation gap and the real exchange rate gap move in the same direction, while the second inequality shows that the net foreign asset gap and the real exchange rate gap move in opposite directions.

The real interest gap is equal to the time derivative of the real exchange rate. Thus, the solution for the real interest gap is given by:

$$r - \bar{r} = \dot{q} = c_1 v_{13} \mu_1 e^{\mu_1 t} + c_2 v_{23} \mu_2 e^{\mu_2 t}$$

Using the same procedure we use for the ratio between the inflation gap and the output gap it is easy to verify that:

$$\lim_{t \to} \frac{r - \bar{r}}{q - \bar{q}} = \mu_1 < 0$$

Hence, the interest rate gap and the real exchange rate gap move in opposite directions when converging to the steady-state.

Figure 1 shows an inflation target change at time zero, announced by the foreign central bank to which the domestic currency is pegged, to be implemented at time T. The inflation target will be reduced from π_o^* to $\pi_1^* < \pi_0^*$. How this economy adjusts to an anticipated and permanent change in the rate of world inflation? We will resort to a phase diagram to answer this question. Figure 2 shows the phase diagram for this experiment. By the Phillips curve we can determine the direction of movement of the inflation rate in each one of the two zones divided by the $\dot{\pi} = 0$ line, as shown in Figure 2. We know that the slope of the dominant eigenvector ray is positive, e.g., when t

becomes large, inflation and output converge to their steady-state values from the same direction ($\lim_{t \to \infty} (\pi - \pi^*)/x = v_{11}/v_{12} > 0$). Thus, when the new inflation target is announced the inflation rate and the output gap jump at time zero to point A. From time zero on the economy follows the path depicted by the arrowed curve in Figure 2.
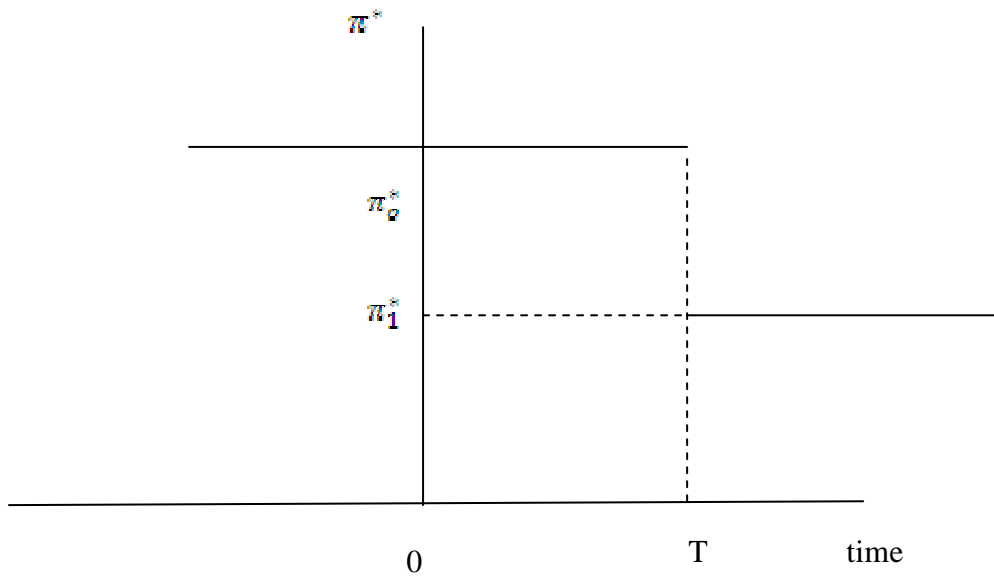

Figure 1

The prediction of this fixed exchange rate regime is that inflation is reduced with transitory output gain. When the inflation target change is implemented at the same time it is announced there is no output gain, since inflation jumps immediately to its new rate.
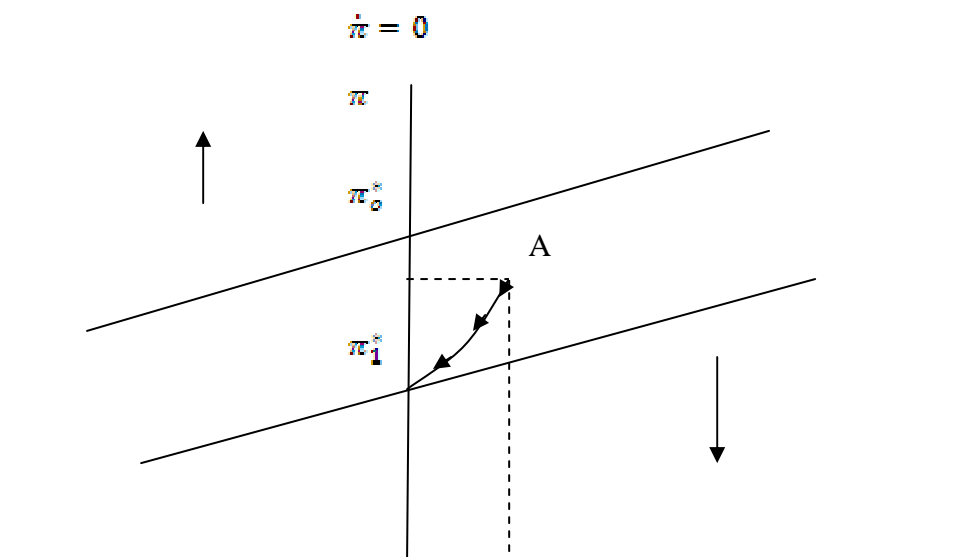

Figure 2

5. Flexible Exchange Rate Regime

The new Keynesian model under a flexible exchange rate system has the same equations as the fixed exchange rate regime for the IS curve, the Phillips curve, the uncovered interest parity and the current account of the balance of payment. The difference is the monetary policy rule. The central bank now controls the interest rate and allows the exchange rate to be market determined. To simplify the algebra we assume a Taylor rule without the output gap term. The initial conditions of the flexible exchange rate system are different from the fixed exchange rate system. The price level and the net foreign asset are predetermined variables. The other variables are free and can jump when there is new information. The specification of the model is given by the following set of equations:

IS: $\quad \dot{x} = \beta_x x + \beta_r (r - \bar{r}) - \beta_a (a - \bar{a}) - \beta_q (q - \bar{q})$

PC: $\quad \dot{\pi} = -\delta x$

UIP: $\quad \dot{q} = r - \bar{r}$

BP: $\quad \dot{a} = \alpha_a (a - \bar{a}) + \alpha_r (r - \bar{r}) + \alpha_q (q - \bar{q})$

MPR: $\quad i = \bar{r} + \pi + \phi(\pi - \bar{\pi})$

IC: $\quad$ Given p(0) and a(0)

The monetary policy rule can be written as:

$$r - \bar{r} = \phi(\pi - \bar{\pi})$$

Substituting the real interest gap of this expression in the three equations, namely the IS curve, the uncovered interest parity and the balance of payments current account, we get, respectively:

$$\dot{x} = \beta_x x + \beta_r \phi(\pi - \bar{\pi}) - \beta_a (a - \bar{a}) - \beta_q (q - \bar{q})$$

$$\dot{q} = \phi(\pi - \bar{\pi})$$

$$\dot{a} = \alpha_a (a - \bar{a}) + \alpha_r \phi(\pi - \bar{\pi}) + \alpha_q (q - \bar{q})$$

The dynamic system of the new Keynesian model, under a flexible exchange rate system, has four differential equations in the following variables: inflation, output gap, real exchange rate and net foreign asset. Net foreign asset is the only predetermined variable. The linear approximation of the dynamic system around the steady state is given by:

$$\begin{bmatrix} \dot{\pi} \\ \dot{x} \\ \dot{s} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} 0 & -\delta & 0 & 0 \\ \beta_r\phi & \beta_x & -\beta_q & -\beta_a \\ \phi & 0 & 0 & 0 \\ \alpha_r\phi & 0 & \alpha_q & \alpha_a \end{bmatrix} \begin{bmatrix} \pi - \bar{\pi} \\ x \\ q - \bar{q} \\ a - \bar{a} \end{bmatrix}$$

The Jacobian of this matrix is given by:

$$J = \begin{bmatrix} 0 & -\delta & 0 & 0 \\ \beta_r\phi & \beta_x & -\beta_q & -\beta_a \\ \phi & 0 & 0 & 0 \\ \alpha_r\phi & 0 & \alpha_q & \alpha_a \end{bmatrix}$$

The determinant of this matrix can be computed using the cofactor expansion along the first line of this matrix, which yields:

$$|J| = (-\delta)(-1)^{1+2} \begin{vmatrix} \beta_r\phi & \beta_q & -\beta_a \\ \phi & 0 & 0 \\ \alpha_r\phi & \alpha_q & \alpha_a \end{vmatrix}$$

Hence,

$$|J| = -\delta\phi\left(\alpha_q\beta_a + \beta_q\alpha_a\right)$$

The trace of this matrix is equal to:

$$tr\ J = \beta_x + \alpha_a$$

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We assume a negative determinant: $|J| < 0$. The dynamic system under this hypothesis has either one negative characteristic root and three positive roots or three positive roots and one negative root. In order to know if there is a solution with three negative roots we have to analyze the characteristic equation of the model. This equation is obtained solving the determinant:

$$\begin{vmatrix} -\mu & -\delta & 0 & 0 \\ \beta_r\phi & \beta_x - \mu & \beta_q & -\beta_a \\ \phi & 0 & -\mu & 0 \\ \alpha_r\phi & 0 & \alpha_q & \alpha_a - \mu \end{vmatrix} = 0$$

Using cofactor expansion along the first line we get the characteristic equation:

$$\mu^4 - (\alpha_a + \beta_x)\mu^3 + (\alpha_a\beta_x + \beta_r\delta\phi)\mu^2 - \delta\phi(\alpha_a\beta_r + \alpha_r\beta_a - \beta_q)\mu + \phi(\alpha_q\beta_a + \alpha_a\beta_q) = 0$$

When the number of changes in sign in the sequence of coefficients [Sydsaeter, Strom and Berck (2000), p.8] a_4, a_3, a_2, a_1, $and\ a_0$ is equal to three there is three positive roots. The sequence is given by:

$$a_4 = 1,\ a_3 = -(\alpha_a + \beta_x), a_2 = (\alpha_a\beta_x + \delta\phi),$$
$$a_1 = -\delta\phi(\alpha_a\beta_r + \alpha_r\beta_a - \beta_q)\ and\ a_o = \phi(\alpha_s\beta_q + \alpha\beta_q)$$

We assume that the coefficients in this sequence are such that there are three positive roots. Hence, the solutions for the inflation rate, the output gap, the real exchange rate and the net foreign asset are, respectively:

$$\pi = \bar{\pi} + c_1 v_{11}\ e^{\mu_1 t}$$
$$x = c_1 v_{12}\ e^{\mu_1 t}$$
$$q = \bar{q} + c_1 v_{13}\ e^{\mu_1 t}$$
$$a = \bar{a} + c_1 v_{14}\ e^{\mu_1 t}$$

where the constant c_1 is obtained from the initial conditions of $a(0)$, μ_1 is the negative characteristic root and the elements of the characteristic vector $v_{i\,j}$ are the solution of the linear system:

$$
\begin{bmatrix}
-\mu & -\delta & 0 & 0 \\
\beta_r\phi & \beta_x-\mu & -\beta_q & -\beta_a \\
\phi & 0 & -\mu & 0 \\
\alpha_r\phi & 0 & \alpha_q & \alpha_a-\mu
\end{bmatrix}
\begin{bmatrix}
v_{i1} \\
v_{i2} \\
v_{i3} \\
v_{i4}
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$

Taking into account the fact that the root μ is negative, the first, the third and the fourth equations imply the following inequalities:

$$
\frac{v_{11}}{v_{12}} = -\frac{\delta}{\mu} > 0
$$

$$
\frac{v_{11}}{v_{13}} = \frac{\mu}{\phi} < 0
$$

$$
\frac{v_{13}}{v_{14}} = -\frac{\alpha_a-\mu}{\alpha_r\mu+\alpha_q} < 0
$$

Dividing the inflation gap by the output gap yields:

$$
\frac{\pi-\bar{\pi}}{x} = \frac{v_{11}}{v_{12}} > 0
$$

Hence, the inflation gap and the output gap move in the same direction. Dividing the inflation gap by the real exchange rate gap we get:

$$
\frac{\pi-\bar{\pi}}{q-\bar{q}} = \frac{v_{11}}{v_{13}} < 0
$$

It follows from this expression that the inflation gap and the real exchange rate gap move in opposite directions. As the real interest rate gap is proportional to the inflation gap according to the monetary policy rule, the real rate of interest gap and the real

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exchange rate gap move in opposite directions. This proposition can also de derived using the UIP condition: $r - \bar{r} = \dot{q}$. It is easy to obtain:

$$r - \bar{r} = c_1 v_{13}\, \mu\, e^{\mu_1 t} = \mu_1 (q - \bar{q}), \mu_1 < 0$$

Dividing the net foreign asset gap by real exchange rate gap we get:

$$\frac{a - \bar{a}}{q - \bar{q}} = \frac{v_{14}}{v_{13}} < 0$$

Hence, the net asset gap and the real exchange rate gap move in the opposite directions when the economy is converging to steady-state.

How this economy adjusts to an anticipated and permanent fall in the inflation target? Figure 1 can be applied to this case as well by changing $\bar{\pi}$ for π^*. The central bank announces at time zero that at time T the inflation target will be reduced from $\bar{\pi}_0$ to $\bar{\pi}_1 < \bar{\pi}_0$. Figure 2 shows the adjustment dynamics since the slope of the dominant eigenvector ray is positive, as the slope of the fixed exchange rate regime. In this flexible exchange rate regime model, inflation can be reduced with a transitory gain in output. If the new inflation target is implemented at the time it is announced, the inflation rate jumps immediately to the new target with no change in output.

Rather than summarizing the results obtained in each policy regime, it is useful to highlight the main difference between the fixed and the flexible exchange rate regime. In the fixed exchange rate regime inflation and the real exchange rate converge to the steady-state from the same direction, e.g., the slope of the dominant eigenvector ray is positive. In the flexible exchange rate regime it is just the opposite, the inflation rate and the real exchange rate converges to the steady-state from opposite directions.

6. Conclusion

The value added of this paper is twofold. Firstly, the paper shows that in a small open economy with perfect access to the world capital market the natural rate of interest is equal to the real international rate of interest. Secondly, it shows that the

representative- agent model is not the proper theoretical framework for a small open economy, but that is not the case with the overlapping generations model.

In the representative-agent framework, the natural rate of interest and the international real rate of interest would be equal only under stringent conditions, namely that the rate of growth of domestic output is equal to the rate of growth of world output. In the infinitely-lived overlapping generations model the natural rate of interest is equal to the real international rate of interest with no strings attached.

The results presented in this paper are a useful conceptual benchmark to measure the natural rate of interest in a small open economy. According to GM (2005) the closed economy representative agent new Keynesian model applies, with minor parameters reinterpretation, to a small open economy. That is not the case with the overlapping generations new Keynesian model presented here since the current account adjustment cannot be washed away.

The rejection of the small open economy representative agent model, based on empirical grounds, has far-reaching implications for monetary policy analysis. Much of the optimal monetary policy literature, as applied by GM and others [see, for example, Woodford (2003)], has used the quadratic loss function as an approximation to the intertemporal utility function of the representative agent as the social welfare criteria to be maximized. This criterion is no longer useful for the small open economy overlapping generations framework since social welfare depends on the weights to be given to each generation, even those that are unborn.

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